

NUMERI COMPLESSI:

1) Calcolare il valore di

a) $w = \frac{z^2 - \bar{z}^2}{(z+1)(\bar{z}-1)+1}$ quando $z = 1 + i$; b) $w = \frac{\overline{(iz)} + |z|}{z+i}$ quando $z = 3 - 4i$;

c) $w = \frac{\bar{z} \cdot z^2 - iz}{2z - 3 - i}$ quando $z = 1 + 2i$; d) $w = \frac{\bar{z}^2 \cdot z - (\bar{iz})^2}{z^2 + 2}$ quando $z = 1 - i$

2) Dati i numeri $v = \beta + 2i$ e $w = 2 + i$, determinare β in modo che risulti $\operatorname{Re} \frac{v}{w} = \operatorname{Im} \frac{v}{w}$.

3) Calcolare le seguenti potenze: a) $(\sqrt{3} - i)^8$; b) $(1 - i\sqrt{3})^8$; c) $w^6 = \left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{6} \right)^6$

4) Calcolare e rappresentare graficamente nel piano di Gauss le radici n-esime (nei casi $n = 3$ e $n = 4$) di: a) $w = 1$; b) $w = i$; c) $-1 - i\sqrt{3}$

5) Determinare le soluzioni in C delle seguenti equazioni:

a) $z^2 - 2iz - 5 = 0$; b) $z + \bar{z} - \frac{1}{\bar{z}} = 0$; c) $|z| = i - 4z$; d) $2z^2 - 4iz - (3 + i\sqrt{3}) = 0$; e) $z^6 = 8z^3$;

f) $z^2 \cdot \bar{z}^2 + 2\bar{z}^2 + iz^2 + 2i = 0$; g) $(2 + 2i)z^4 - i\sqrt{2}z = 0$; h) $z^6 + 4z^3 + 8 = 0$; i) $(2z + 3)^3 = -27i$;

l) $z^2 = \bar{z}$; m) $\left(\frac{-}{z}\right)^3 \cdot z^4 = -2z^2$; n) $z^3 = \left(\frac{-}{z}\right)^2$.

6) Determinare le soluzioni in C dei seguenti sistemi di equazioni:

a) $\begin{cases} z \cdot \bar{w} = i \\ |z|^2 \cdot w + z = 1 \end{cases}$; b) $\begin{cases} \left|\frac{w}{2}\right| = |z| = \sqrt{2} \\ 2z + w = 4 \end{cases}$; c) $\begin{cases} |z - (1 + 3i)| = 2 \\ i \cdot (z + \bar{z}) = z - \bar{z} \end{cases}$; d) $\begin{cases} z + \bar{z} = -\frac{8}{5}|z| \\ \operatorname{Im} z = 2 + \operatorname{Re} z \end{cases}$

e) $\begin{cases} z^2 = -|w|^4 \\ w^2 - (2i + 1)w = \frac{z + \bar{z}}{3} + 1 - i \end{cases}$; f) $\begin{cases} z^2 + w = 1 \\ 2\bar{w} + iz = 2 \end{cases}$; g) $\begin{cases} z \cdot (1 + |w|) = -2i \\ w \cdot \bar{z} + (2 - iz) \cdot |w| = |z| \cdot w \end{cases}$

h) $\begin{cases} \left(\frac{-}{z}\right)^2 - w^2 = -1 \\ \left(\frac{-}{w}\right)^2 - z = 0 \end{cases}$; i) $\begin{cases} z + w = 1 + i \\ \left(\frac{-}{w}\right)^2 + \bar{z} = 1 - i \end{cases}$; l) $\begin{cases} \bar{w} \cdot z + 1 - i\sqrt{3} = 0 \\ z^3 \cdot \left(\frac{-}{z}\right)^2 - \left(\frac{-}{z}\right)^2 \cdot w = 0 \end{cases}$; m) $\begin{cases} (w + |z|) \cdot (\bar{i}w + w) = 0 \\ 2|z| + w + 3i = 3 + z + \bar{z} \\ |z| = |w| \end{cases}$

RISULTATI: 1a) $-1 + i$; 1b) $2 + i$; 1c) $2 - 3i$, 1d) $-1/2 + 3/2 i$; 2) $\beta=2/3$; 3a) $2^7(-1 + i\sqrt{3})$; 3b) $-2^7(1 + i\sqrt{3})$;

3c) $\frac{i}{8}$; 4a) con $n=3$: $z_0=1$, $z_{1,2}=-\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$, con $n=4$: $z_0=1$, $z_1=i$, $z_2=-1$, $z_3=-i$; 4b) con $n=3$:

$$z_{0,1}=\pm\frac{\sqrt{3}}{2}+i\frac{1}{2}, z_2=-i, \text{ con } n=4: z_{0,1,2,3}=\cos\left(\frac{\pi}{8}+k\frac{\pi}{2}\right)+isen\left(\frac{\pi}{8}+k\frac{\pi}{2}\right), k=0,1,2,3; 4c) \dots;$$

$$5a) z_{2,1}=2+i, z_2=-2+i; 5b) z_1=\frac{\sqrt{2}}{2}, z_2=-\frac{\sqrt{2}}{2}; 5c) z=-\frac{\sqrt{15}}{60}+\frac{1}{4}i; 5d) z_1=\frac{\sqrt{3}}{2}+\frac{3}{2}i, z_2=-\frac{\sqrt{3}}{2}+\frac{1}{2}i; 5e) z_{1,2,3}=0, z_4=2, z_{5,6}=-1\pm i\sqrt{3}; 5f) z_{1,2}=\pm i\sqrt{2}, z_{3,4}=\pm\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right); 5g) z_1=0, z_{2,3,4}=$$

$$\sqrt[3]{\frac{1}{2}}\left(\cos\left(\frac{\pi}{12}+\frac{2k\pi}{3}\right)+isen\left(\frac{\pi}{12}+\frac{2k\pi}{3}\right)\right), k=0,1,2; 5h) |z|=\sqrt{2}, \alpha_1=\pi/4, \alpha_2=5\pi/12, \alpha_3=11\pi/12, \alpha_4=13\pi/12,$$

$$\alpha_5=19\pi/12, \alpha_6=7\pi/4; 5i) z_1=-\frac{3}{2}+\frac{3}{2}i, z_2=\frac{-3\sqrt{3}-6}{4}-\frac{3}{4}i, z_3=\frac{3\sqrt{3}-6}{4}-\frac{3}{4}i; 5l) z_1=0, z_2=1, z_{3,4}=$$

$$-\frac{1}{2}\pm i\frac{\sqrt{3}}{2}; 5m) z_1=0, z_2=-\sqrt[5]{2}; 5n) z=0 \text{ e } z=\text{radici quinte dell'unit\`a}; 6a) z=1/2+1/2i, w=1-i;$$

$$6b) z_1=1+i \text{ e } w_1=2-2i, z_2=1-i \text{ e } w_2=2+2i; 6c) z_1=1+i, z_2=3+3i; 6d) z_1=-8-6i, z_2=-8/7+6/7i;$$

$$6e) w_1=i \text{ e } z_1=\pm i, w_2=1+i \text{ e } z_2=\pm 2i; 6f) w_1=5/4 \text{ e } z_1=1/2i, w_2=1 \text{ e } z_2=0, w_3=\frac{7}{8}+\frac{\sqrt{3}}{8}i \text{ e } z_3=$$

$$\frac{\sqrt{3}}{4}-\frac{1}{4}i, w_4=\frac{7}{8}-\frac{\sqrt{3}}{8}i \text{ e } z_4=-\frac{\sqrt{3}}{4}-\frac{1}{4}i; 6g) z=-2i \text{ e } w=0, z=\frac{-2i}{1+\sqrt{2}} \text{ e } w=1+i; 6h) z_1=\frac{1}{2}-i\frac{\sqrt{3}}{2} \text{ e }$$

$$w_1=\frac{\sqrt{3}}{2}+\frac{1}{2}i, z_2=\frac{1}{2}-i\frac{\sqrt{3}}{2} \text{ e } w_2=-\frac{\sqrt{3}}{2}-\frac{1}{2}i, z_3=\frac{1}{2}+i\frac{\sqrt{3}}{2} \text{ e } w_3=-\frac{\sqrt{3}}{2}+\frac{1}{2}i, z_4=\frac{1}{2}+i\frac{\sqrt{3}}{2} \text{ e }$$

$$w_4=\frac{\sqrt{3}}{2}-\frac{1}{2}i; 6i) z_1=1+i \text{ e } w_1=0, z_2=i \text{ e } w_2=1; 6l) z_1=\frac{\sqrt[4]{2}}{2}(1+i\sqrt{3}) \text{ e } w_1=\frac{\sqrt[4]{8}}{2}(1-i\sqrt{3}),$$

$$z_2=\frac{\sqrt[4]{2}}{2}(-\sqrt{3}+i) \text{ e } w_2=\frac{\sqrt[4]{8}}{2}(\sqrt{3}+i), z_3=\frac{\sqrt[4]{2}}{2}(-1-i\sqrt{3}) \text{ e } w_3=\frac{\sqrt[4]{8}}{2}(-1+i\sqrt{3}), z_4=\frac{\sqrt[4]{2}}{2}(\sqrt{3}-i) \text{ e }$$

$$w_4=\frac{\sqrt[4]{8}}{2}(-\sqrt{3}-i); 6m) z=3\sqrt{2} \text{ e } w=3-3i.$$