NUMERI COMPLESSI:

1) Calcolare il valore di

a) $w = \dfrac{z^2 - \bar{z}^2}{(z+1)(\bar{z}-1)+1}$ quando z = 1 + i ; b) $w = \dfrac{\overline{(iz)} + |z|}{z+i}$ quando z = 3 – 4i ;

c) $w = \dfrac{\bar{z} \cdot z^2 - iz}{2z - 3 - i}$ quando z = 1 + 2i ; d) $w = \dfrac{\bar{z}^2 \cdot z - (\overline{iz})^2}{z^2 + 2}$ quando z = 1 – i

2) Dati i numeri v = β + 2i e w = 2 + i, determinare β in modo che risulti $\mathrm{Re}\dfrac{v}{w} = \mathrm{Im}\dfrac{v}{w}$.

3) Calcolare le seguenti potenze: a) $\left(\sqrt{3} - i\right)^8$; b) $\left(1 - i\sqrt{3}\right)^8$; c) $w^6 = \left(\cos\dfrac{\pi}{3} - i sen\dfrac{\pi}{6}\right)^6$

4) Calcolare e rappresentare graficamente nel piano di Gauss le radici n-esime (nei casi n = 3 e n = 4) di: a) w = 1; b) w = i ; c) $-1 - i\sqrt{3}$

5) Determinare le soluzioni in C delle seguenti equazioni:

a) $z^2 - 2iz - 5 = 0$; b) $z + \bar{z} - \dfrac{1}{\bar{z}} = 0$; c) $|z| = i - 4z$; d) $2z^2 - 4iz - (3 + i\sqrt{3}) = 0$; e) $z^6 = 8z^3$;
f) $z^2 \cdot \bar{z}^2 + 2\bar{z}^2 + iz^2 + 2i = 0$; g) $(2+2i)z^4 - i\sqrt{2}z = 0$; h) $z^6 + 4z^3 + 8 = 0$; i) $(2z+3)^3 = -27i$;

l) $z^2 = \bar{z}$; m) $(\bar{z})^3 \cdot z^4 = -2z^2$; n) $z^3 = (\bar{z})^2$.

6) Determinare le soluzioni in C dei seguenti sistemi di equazioni:

a) $\begin{cases} z \cdot \bar{w} = i \\ |z|^2 \cdot w + z = 1 \end{cases}$; b) $\begin{cases} \left|\dfrac{w}{2}\right| = |z| = \sqrt{2} \\ 2z + w = 4 \end{cases}$; c) $\begin{cases} |z - (1+3i)| = 2 \\ i \cdot (z + \bar{z}) = z - \bar{z} \end{cases}$; d) $\begin{cases} z + \bar{z} = -\dfrac{8}{5}|z| \\ \mathrm{Im}\, z = 2 + \mathrm{Re}\, z \end{cases}$

e) $\begin{cases} z^2 = -|w|^4 \\ w^2 - (2i+1)w = \dfrac{z+\bar{z}}{3} + 1 - i \end{cases}$; f) $\begin{cases} z^2 + w = 1 \\ 2\bar{w} + iz = 2 \end{cases}$; g) $\begin{cases} z \cdot (1 + |w|) = -2i \\ w \cdot \bar{z} + (2 - iz) \cdot |w| = |z| \cdot w \end{cases}$

h) $\begin{cases} (\bar{z})^2 - w^2 = -1 \\ (\bar{w})^2 - z = 0 \end{cases}$; i) $\begin{cases} z + w = 1 + i \\ (|w|)^2 + \bar{z} = 1 - i \end{cases}$; l) $\begin{cases} \bar{w} \cdot z + 1 - i\sqrt{3} = 0 \\ z^3 \cdot (|z|)^2 - (\bar{z})^2 \cdot w = 0 \end{cases}$; m) $\begin{cases} (w + |z|) \cdot (i\bar{w} + w) = 0 \\ 2|z| + w + 3i = 3 + z + \bar{z} \\ |z| = |w| \end{cases}$

RISULTATI: 1a) -1 + i ; 1b) 2 + i ; 1c) 2 – 3i , 1d) -1/2 + 3/2 i ; 2) $\beta=2/3$; 3a) $2^7\left(-1+i\sqrt{3}\right)$; 3b) $-2^7\left(1+i\sqrt{3}\right)$; 3c) $\frac{i}{8}$; 4a) con n = 3: $z_0 = 1$, $z_{1,2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$, con n = 4: $z_0 = 1$, $z_1 = i$, $z_2 = -1$, $z_3 = -i$; 4b) con n = 3:

$z_{0,1} = \pm\frac{\sqrt{3}}{2} + i\frac{1}{2}$, $z_2 = -i$, con n = 4 : $z_{0,1,2,3} = \cos\left(\frac{\pi}{8}+k\frac{\pi}{2}\right)+i sen\left(\frac{\pi}{8}+k\frac{\pi}{2}\right), k = 0,1,2,3$; 4c) …;

5a) $z2_1 = 2 + i$, $z_2 = -2 + i$; 5b) $z_1 = \frac{\sqrt{2}}{2}$, $z_2 = -\frac{\sqrt{2}}{2}$; 5c) $z = -\frac{\sqrt{15}}{60}+\frac{1}{4}i$; 5d) $z_1 = \frac{\sqrt{3}}{2}+\frac{3}{2}i$, $z_2 = -\frac{\sqrt{3}}{2}+\frac{1}{2}i$; 5e) $z_{1,2,3}= 0$, $z_4 = 2$, $z_{5,6} = -1 \pm i\sqrt{3}$; 5f) $z_{1,2} = \pm i\sqrt{2}$, $z_{3,4} = \pm\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)$; 5g) $z_1 = 0$, $z_{2,3,4} = \sqrt[3]{\frac{1}{2}}\left(\cos\left(\frac{\pi}{12}+\frac{2k\pi}{3}\right)+i sen\left(\frac{\pi}{12}+\frac{2k\pi}{3}\right)\right), k = 0,1,2$; 5h) $|z|=\sqrt{2}$, $\alpha_1 = \pi/4$, $\alpha_2 = 5\pi/12$, $\alpha_3=11\pi/12$, $\alpha_4=13\pi/12$, $\alpha_5=19\pi/12$, $\alpha_6=7\pi/4$; 5i) $z_1 = -\frac{3}{2}+\frac{3}{2}i$, $z_2 = \frac{-3\sqrt{3}-6}{4}-\frac{3}{4}i$, $z_3 = \frac{3\sqrt{3}-6}{4}-\frac{3}{4}i$; 5l) $z_1 = 0$, $z_2 = 1$, $z_{3,4} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$; 5m) $z_1 = 0$, $z_2 = -\sqrt[5]{2}$; 5n) z = 0 e z = radici quinte dell'unità ; 6a) z = 1/2 + 1/2i , w = 1 – i ;

6b) $z_1= 1 + i$ e $w_1 = 2 – 2i$, $z_2 = 1 – i$ e $w_2 = 2 + 2i$; 6c) $z_1 = 1 + i$, $z_2 = 3 + 3i$; 6d) $z_1 = -8 – 6i$, $z_2 = -8/7 + 6/7 i$;

6e) $w_1 = i$ e $z_1 = \pm i$, $w_2 = 1 + i$ e $z_2 = \pm 2i$; 6f) $w_1 = 5/4$ e $z_1 = 1/2\ i$, $w_2 = 1$ e $z_2 = 0$, $w_3 = \frac{7}{8}+\frac{\sqrt{3}}{8}i$ e $z_3 = \frac{\sqrt{3}}{4}-\frac{1}{4}i$, $w_4 = \frac{7}{8}-\frac{\sqrt{3}}{8}i$ e $z_4 = -\frac{\sqrt{3}}{4}-\frac{1}{4}i$; 6g) z = -2i e w = 0 , $z = \frac{-2i}{1+\sqrt{2}}$ e w = 1 + i ; 6h) $z_1 = \frac{1}{2}-i\frac{\sqrt{3}}{2}$ e $w_1 = \frac{\sqrt{3}}{2}+\frac{1}{2}i$, $z_2 = \frac{1}{2}-i\frac{\sqrt{3}}{2}$ e $w_2 = -\frac{\sqrt{3}}{2}-\frac{1}{2}i$, $z_3 = \frac{1}{2}+i\frac{\sqrt{3}}{2}$ e $w_3 = -\frac{\sqrt{3}}{2}+\frac{1}{2}i$, $z_4 = \frac{1}{2}+i\frac{\sqrt{3}}{2}$ e $w_4 = \frac{\sqrt{3}}{2}-\frac{1}{2}i$; 6i) $z_1 = 1 + i$ e $w_1 = 0$, $z_2 = i$ e $w_2 = 1$; 6l) $z_1 = \frac{\sqrt[4]{2}}{2}\left(1+i\sqrt{3}\right)$ e $w_1 = \frac{\sqrt[4]{8}}{2}\left(1-i\sqrt{3}\right)$,

$z_2 = \frac{\sqrt[4]{2}}{2}\left(-\sqrt{3}+i\right)$ e $w_2 = \frac{\sqrt[4]{8}}{2}\left(\sqrt{3}+i\right)$, $z_3 = \frac{\sqrt[4]{2}}{2}\left(-1-i\sqrt{3}\right)$ e $w_3 = \frac{\sqrt[4]{8}}{2}\left(-1+i\sqrt{3}\right)$, $z_4 = \frac{\sqrt[4]{2}}{2}\left(\sqrt{3}-i\right)$ e

$w_4 = \frac{\sqrt[4]{8}}{2}\left(-\sqrt{3}-i\right)$; 6m) $z = 3\sqrt{2}$ e w = 3 – 3i .