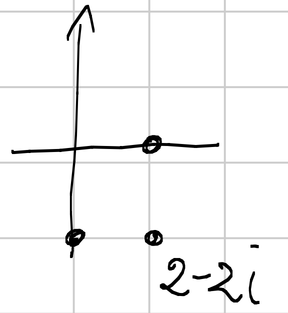
Lezione 10 - Analisi Matematica 1 - 21 ottobre 2012

Esercizi

1) Calcolare $\sqrt{2-2i}$ e $\sqrt[4]{1+i\sqrt{3}}$

$\sqrt{2-2i}$ $\quad w = 2-2i \quad |w| = \sqrt{4+4} = 2\sqrt{2}$



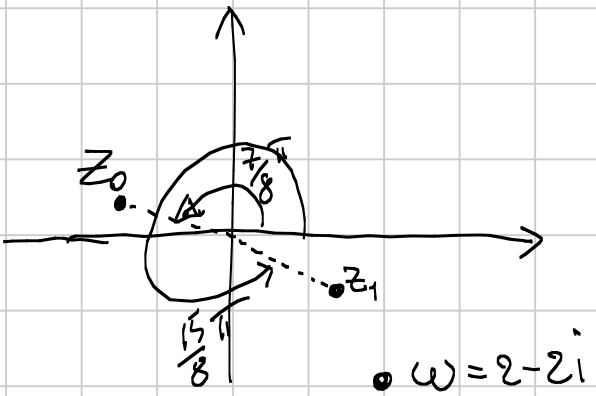
$$\arg(w) = 2\pi + \operatorname{arctg}\frac{-2}{2}$$

$$= 2\pi - \operatorname{arctg} 1 = 2\pi - \frac{\pi}{4} = \frac{7}{4}\pi$$

$$\sqrt{2\sqrt{2}\left(\cos\left(\frac{7}{4}\pi\right) + i \operatorname{sen}\left(\frac{7}{4}\pi\right)\right)}$$

$$z_0 = \sqrt[4]{8}\cdot\left(\cos\left(\frac{7}{4}\pi\cdot\frac{1}{2}\right) + i \operatorname{sen}\left(\frac{7}{4}\pi\cdot\frac{1}{2}\right)\right) = \sqrt[4]{8}\cdot\left(\cos\frac{7}{8}\pi + i \operatorname{sen}\frac{7}{8}\pi\right)$$

$$z_1 = \sqrt[4]{8}\left(\cos\left(\frac{7}{8}\pi + \frac{2\pi}{2}\right) + i \operatorname{sen}\left(\frac{7}{8}\pi + \frac{2\pi}{2}\right)\right) = \sqrt[4]{8}\left(\cos\frac{15}{8}\pi + i \operatorname{sen}\frac{15}{8}\pi\right)$$



$$\cos\frac{7\pi}{8} = \cos\left(\pi - \frac{\pi}{8}\right)$$

$$= \cos\pi\cdot\cos\frac{\pi}{8} + \operatorname{sen}\pi \operatorname{sen}\frac{\pi}{8}$$

$$= -\cos\frac{\pi}{8}$$

$$= -\cos\left(\frac{\pi/4}{2}\right)$$

$$\sqrt[4]{1+i\sqrt{3}}$$

$$w = 1+i\sqrt{3} \qquad |w| = \sqrt{1+3} = 2 \qquad \arg(w) = \operatorname{arcTg}\frac{\sqrt{3}}{1} = \pi/3$$

$$= 2\left(\cos\frac{\pi}{3} + i\,\mathrm{sen}\frac{\pi}{3}\right)$$

$$z_0 = \sqrt[4]{2}\left(\cos\left(\frac{\pi/3}{4}\right) + i\,\mathrm{sen}\frac{\pi}{12}\right)$$

$$z_1 = \sqrt[4]{2}\left(\cos\left(\frac{\pi}{12}+\frac{\pi}{2}\right) + i\,\mathrm{sen}\left(\frac{\pi}{12}+\frac{\pi}{2}\right)\right)$$

$$z_2 = \sqrt[4]{2}\left(\cos\left(\frac{\pi}{12}+\pi\right) + i\,\mathrm{sen}\left(\frac{\pi}{12}+\pi\right)\right)$$

$$z_3 = \sqrt[4]{2}\left(\cos\left(\frac{\pi}{12}+\frac{3}{2}\pi\right) + i\,\mathrm{sen}\left(\frac{\pi}{12}+\frac{3}{2}\pi\right)\right)$$

$$\mathrm{sen}\left(\frac{\pi}{12}\right) = \mathrm{sen}\left(\frac{\pi/6}{2}\right)$$

Formule Bisezione

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi/6}{2}\right)$$

4) Calcolare le radici di $z + i\bar{z}^2 = -2i$

dim

$z = x+iy$ $\quad x+iy+i(x-iy)^2+2i=0$

$x+iy+i(x^2-y^2-2ixy)+2i=0$

$x+2xy+i(y+x^2-y^2+2)=0$

$\Updownarrow$

$$\begin{cases} x(1+2y)=0 \\ x^2=y^2-y-2 \end{cases} \iff \begin{cases} x=0 \\ y^2-y-2=0 \end{cases} \text{ o } \cancel{\begin{cases} y=-\frac{1}{2} \\ x^2=\frac{1}{4}-\frac{1}{2}-2 \end{cases}}$$

non ha sol. reali

↑ Questo è un sistema Reale
$$\begin{cases} (\text{Parte Reale})=0 \\ (\text{Parte Immaginaria})=0 \end{cases}$$

$$\iff \begin{cases} x=0 \\ y_{1,2}=\dfrac{1\pm\sqrt{1+8}}{2} \end{cases}$$

$$\boxed{\begin{aligned} z_1 &= 2i \\ z_2 &= -i \end{aligned}}$$

x casa
Verificarle

6) Calcolare le radici complesse di $z^2 + z\bar{z} = 1 + 2i$

$z = x + iy$ $\left(z_1 = \frac{1}{\sqrt{2}} + \sqrt{2}\, i \quad z_2 = -\frac{1}{\sqrt{2}} - \sqrt{2}\, i \right)$

(Da fare!)

7) Sia $w = \dfrac{\bar{z}(z+i) - \bar{z}^2}{z^2 - 1}$. Se $z = 1 - 2i$, quale tra le seguenti risposte è VERA?

A) $\mathrm{Re}\, w > \mathrm{Im}\, w$ NO B) Nessuna delle altre risposte è vera NO

C) $\mathrm{Re}\, w < 0$ SÌ D) $\mathrm{Im}\, w = 0$ NO

$$w = \frac{\bar{z} \cdot z + i\bar{z} - \bar{z}^2}{z^2 - 1} = \frac{|z|^2 + i\bar{z} - \bar{z}^2}{z^2 - 1} \qquad z = 1 - 2i$$

$$= \frac{|1-2i|^2 + i(1+2i) - (1+2i)^2}{(1-2i)^2 - 1}$$

$$= \frac{5 + i - 2 - (1 - 4 + 4i)}{\cancel{1} - 4 - 4i - \cancel{1}} = \frac{5 - 2 + 3 + i(1-4)}{-4(1+i)}$$

$$= -\frac{6 - 3i}{1+i} \cdot \frac{1}{4} = -\frac{3}{4} \frac{2-i}{1+i} \frac{1-i}{1-i} = -\frac{3}{4} \frac{2 - i - 2i - 1}{2}$$

$$= -\frac{3}{8}(1 - 3i) = -\frac{3}{8} + \frac{9}{8} i$$

8) Determinare tutte le soluzioni in $\mathbb{C}$ del seguente sistema

$$\begin{cases} 3z - i|w| = i + \bar{z} \\ w - 2|z| = 4i \end{cases}$$

Sostituendo $z = a+ib$ e $w = c+id$ trovo un sistema di 4 grado, quando ho eliminato le radici

$$\begin{cases} 2z = i|w| + i - (z - \bar{z}) \\ // \end{cases} \qquad \begin{cases} 2z = i|w| + i - 2i\,\mathrm{Im}\, z \\ // \end{cases}$$

$$\begin{cases} 2z = i\left(|w| + 1 - 2\,\mathrm{Im}\, z\right) \\ // \end{cases} \leftarrow$$ Mi dice che $\mathrm{Re}\, z = 0$

$z = ib$, ovvero $\boxed{a = 0}$

$$\begin{cases} 2\cancel{i}b = \cancel{i}\left(|w| + 1 - 2b\right) \\ w - 2|b| = 4i \end{cases} \iff \begin{cases} 4b = |w| + 1 \\ // \end{cases}$$

$$\begin{cases} 2b = \dfrac{|w|+1}{2} > 0 \\ w - \dfrac{|w|+1}{2} = 4i \end{cases} \qquad \begin{cases} 2b = \dfrac{|w|+1}{2} \\ 2(c+id) - \sqrt{c^2+d^2} - 1 - 8i = 0 \end{cases}$$

$$\begin{cases} // \\ 2c - 1 = \sqrt{c^2+d^2} \\ 2d = 8 \end{cases} \qquad \begin{cases} // \\ 4c^2 + 1 - 4c = c^2 + 16 \\ d = 4 \end{cases}$$

$$\begin{cases} b = \dfrac{|w|+1}{4} \\ d = 4 \\ 3c^2 - 4c - 15 = 0 \end{cases}$$

$$c_{1,2} = \frac{2 \pm \sqrt{4+45}}{3}$$

$$c_1 = 3$$

$$c_2 = -5/3$$

$$\begin{cases} b = 3/2 \\ c_1 = 3 \\ d = 4 \end{cases}$$

$$b = \frac{1}{4}\left(1 + \left|-\frac{5}{3} + 4i\right|\right) = \frac{1}{4}\left(1 + \frac{13}{3}\right) = \frac{4}{3}$$

$$c_2 = -5/3$$

$$d = 4$$

$$(z_1, w_1) = \left(\frac{3}{2}i \, ; \, 3+4i\right) \qquad (z_2, w_2) = \left(\frac{4}{3}i \, ; \, -\frac{5}{3} + 4i\right)$$

13) Calcolare tutte le radici complesse di

$$\begin{cases} \left|\frac{w}{2}\right| = |z| = \sqrt{2} \\ 2z + w = 4 \end{cases}$$

$a+ib = z \quad c+id = w$

$$\begin{cases} \left|\frac{w}{2}\right| = |z| \\ |z| = \sqrt{2} \\ 2z+w=4 \end{cases} \quad \begin{cases} |w| = 2|z| \\ \sqrt{a^2+b^2} = \sqrt{2} \\ w = 4-2z \end{cases} \quad \begin{cases} |4-2z| = 2|z| \\ a^2+b^2 = 2 \\ w = 4-2z \end{cases}$$

$w = c+id \qquad z = a+ib$

$$\begin{cases} |4-2(a+ib)| = 2|a+ib| \\ a^2+b^2 = 2 \\ c+id = 4-2a-2ib \end{cases}$$

$$\begin{cases} (4-2a)^2 + 4b^2 = 4(a^2+b^2) \\ a^2+b^2 = 2 \\ c = 4-2a \\ d = -2b \end{cases}$$

$$\begin{cases} 16 + \cancel{4a^2} - 16a + \cancel{4b^2} = \cancel{4a^2} + \cancel{4b^2} \\ // \end{cases} \qquad \begin{cases} a = 1 \\ b^2 = 1 \\ c = 2 \\ d = -2b \end{cases}$$

$$\begin{cases} a=1 \\ c=2 \\ b=1 \\ d=-2 \end{cases} \qquad \begin{cases} a=1 \\ c=2 \\ b=-1 \\ d=2 \end{cases}$$

$$(z_1, w_1) = (1+i, 2-2i) \qquad (z_2, w_2) = (1-i, 2+2i)$$