Lezione 10 - Analisi Matematica 1 - 21 ottobre 2012

2) Calcolare le radici in $\mathbb{C}$ di "$z^2 \cdot |z|^2 = i$"

$$\left(R: \ z_1 = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, \ z_2 = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$$

$$z^2 \cdot |z|^2 = i \Rightarrow \left|z^2 \cdot |z|^2\right| = |i|$$

$$|z^2| \cdot |z|^2 = 1 \iff |z|^4 = 1 \iff |z| = 1$$

L'eq. di potenza diventa

$$z^2 \cdot 1^2 = i \iff z^2 = i = \left(\cos\frac{\pi}{2} + i \operatorname{sen}\frac{\pi}{2}\right)$$

$$z_1 = \left(\cos\frac{\pi}{4} + i \operatorname{sen}\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$

$$z_2 = \left(\cos\left(\frac{\pi}{4}+\pi\right) + i \operatorname{sen}\left(\frac{\pi}{4}+\pi\right)\right) = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$$

3) Calcolare le radici in $\mathbb{C}$ di "$z^4 = |z^2| + 2$"

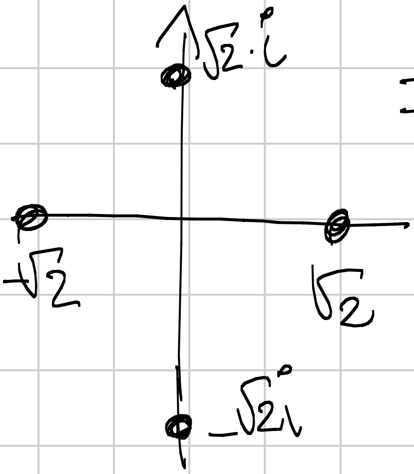
$$|z^4| = \big||z|^2+2\big| \iff |z|^4 = |z|^2+2$$

$$\iff t=|z|^2 \quad t^2-t-2=0 \iff t=|z|^2 \quad t_{1,2}=\frac{1\pm\sqrt{1+8}}{2}$$

$$\iff |z|^2=2 \qquad \cancel{|z|^2=-1}$$

NON ACCETTABILE ($|z|^2 \geq 0$!!)

$$\Rightarrow \boxed{|z|^2=2} \Rightarrow z^4 = 4 = 4(\cos 0 + i \operatorname{sen} 0) \Rightarrow$$



$$\Rightarrow \begin{cases} z_0 = \sqrt{2}(\cos 0 + i \operatorname{sen} 0) = \sqrt{2} \\ z_1 = \sqrt{2}\left(\cos\left(0+\frac{2\pi}{4}\right) + i \operatorname{sen}\left(0+\frac{2\pi}{4}\right)\right) = i\sqrt{2} \\ z_2 = \sqrt{2}\left(\cos\left(0+\frac{2}{4}\cdot 2\pi\right) + i \operatorname{sen}\left(0+\frac{2}{4}\cdot 2\pi\right)\right) = -\sqrt{2} \\ z_3 = \sqrt{2}\left(\cos\left(0+\frac{3}{4}\cdot 2\pi\right) + i \operatorname{sen}\left(0+\frac{3}{4}\cdot 2\pi\right)\right) = -\sqrt{2}i \end{cases}$$

4) $z + i\bar{z}^2 = -2i$

$z = x + iy$ $\quad x + iy + i(x - iy)^2 + 2i = 0$

$$x + iy + i(x^2 - y^2 - 2ixy) + 2i = 0$$

$$x + 2xy + i(y + x^2 - y^2 + 2) = 0$$

$\Updownarrow$

sistema Reale

$$\begin{cases} x(1+2y) = 0 & \leftarrow \text{Parte Reale} = 0 \\ x^2 = y^2 - y - 2 & \leftarrow \text{Parte Imag} = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y^2 - y - 2 = 0 \end{cases} \quad \text{o} \quad \cancel{\begin{cases} y = -\frac{1}{2} \\ x^2 = \frac{1}{4} - \frac{1}{2} - 2 \end{cases}}$$

non ha sol. reali

$$\Leftrightarrow \begin{cases} x = 0 \\ y_{1,2} = \dfrac{1 \pm \sqrt{1+8}}{2} \end{cases}$$

$$\boxed{\begin{aligned} z_1 &= 2i \\ z_2 &= -i \end{aligned}}$$

x casa

Verificare

5) Calcolare le radici complesse di "$z^3\bar{z}+3z^2-4=0$"

$z=x+iy$ NON CONVIENE

$z^2\cdot z\cdot\bar{z}+3z^2-4=0$

$z^2\cdot|z|^2+3z^2-4=0$

$z^2(|z|^2+3)=4$ $\qquad z^2=\dfrac{4}{|z|^2+3}$

$\Downarrow$

$|z|^2=\dfrac{4}{|z|^2+3}$

$|z|^4+3|z|^2=4 \Rightarrow |z|^4+3|z|^2-4=0$

$(|z|^2+4)(|z|^2-1)=0$ ma $|z|^2=-4$ NON è accettabile

$\Rightarrow \boxed{|z|^2=1}$

$z^2\cdot|z|^2+3z^2-4=0$

$z^2+3z^2=4 \Leftrightarrow 4z^2=4 \Leftrightarrow z^2=1$

$\Leftrightarrow z_{1,2}=\pm1$ $\qquad \boxed{z_1=1 \quad z_2=-1}$

10) Calcolare tutte le radici complesse di

$$\begin{cases} i\bar{z}\,w = \bar{w}^2 \\ i z \bar{w} = z^2 \\ |w| = 2 \end{cases}$$

(Non fatto a lezione)

dim

$$\begin{cases} w^2 = -i z \bar{w} \\ z^2 = i z \bar{w} \\ |w| = 2 \end{cases} \qquad \begin{cases} w^2 = -z^2 \\ / \\ / \end{cases}$$

$$\begin{cases} w = i z \\ -z^2 = -i z(-i\bar{z}) \\ |w| = 2 \end{cases} \quad \text{o} \quad \begin{cases} w = -i z \\ -z^2 = -i z(i\bar{z}) \\ |w| = 2 \end{cases}$$

$$\begin{cases} w = i z \\ z^2 = |z|^2 \\ |w| = 2 = |z| \end{cases} \quad \text{o} \quad \begin{cases} w = -i z \\ z^2 = -|z|^2 \\ |w| = 2 = |z| \end{cases}$$

$-i(\pm i\sqrt{2})$

$$\begin{cases} z = \pm\sqrt{2} \\ w = \pm i\sqrt{2} \end{cases} \qquad \begin{cases} z = \pm i\sqrt{2} \\ w = \pm\sqrt{2} \end{cases}$$

11) Calcolare Tutte le radici di (Non fatto a lezione)

$$\begin{cases} z^3 = z \\ |w| = 3|z| \\ \bar{z}w + z\bar{w} = 2 \end{cases}$$

dim

$z^3 = z \Rightarrow |z|^3 = |z| \Leftrightarrow \cancel{|z| = 0}$ NON ACCETT.

$|z| = 1$

$\Longrightarrow |w| = 3$

$z \neq 0 \quad z^3 = z \Leftrightarrow z^2 = 1 \Leftrightarrow z = \pm 1$

$\boxed{z = 1} \quad \begin{cases} |w| = 3 \\ w + \bar{w} = 2\mathrm{Re}\, w = 2 \end{cases} \quad \begin{cases} 1 + b^2 = 9 \\ \mathrm{Re}\, w = 1 \end{cases} \quad w = a + ib$

$\begin{cases} b = \pm 2\sqrt{2} \\ a = 1 \end{cases} \quad w_{1,2} = 1 \pm 2\sqrt{2}i$

$(z, w) = (1, 1 \pm 2\sqrt{2}i)$

$z = -1 \begin{cases} |w| = 3 \\ -w - \bar{w} = -2\mathrm{Re}\, w = 2 \end{cases} \quad \begin{cases} 1 + b^2 = 9 \\ \mathrm{Re}\, w = a = -1 \end{cases} \quad \begin{cases} b = \pm 2\sqrt{2}i \\ a = -1 \end{cases}$

$(z, w) = (-1, -1 \pm 2\sqrt{2}i)$

12) Determinare $z \in \mathbb{C}$ t.c. $\operatorname{Im} z \leq 0$ e la radice quadrata di z coincide con $2\bar{z}$

(Non fatto a lezione)

dim

$$\begin{cases} \operatorname{Im} z \leq 0 \\ z = (2\bar{z})^2 \end{cases} \qquad z = a + ib$$

$$\begin{cases} \operatorname{Im} z \leq 0 \\ a + ib = 4(a^2 - b^2 - 2iab) \end{cases} \qquad \begin{cases} \operatorname{Im} z \leq 0 \\ -8b = b \\ 4a^2 - 4b^2 - a = 0 \end{cases}$$

$$\begin{cases} b = 0 \\ a(4a - 1) = 0 \end{cases} \qquad \begin{aligned} z_1 &= 0 \\ z_2 &= 1/4 \end{aligned}$$