Lezione n.ro 2 Analisi 1 7 ottobre 2013

Titolo nota 29/09/2012

MOLTO IMPORTANTE

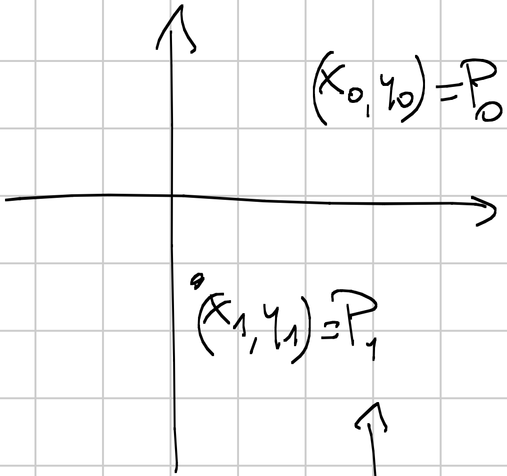
$\boxed{\text{NON È POSSIBILE}}$

STUDIARE TUTTI I RISULTATI DI ANALISI 1 NELL'INTEVALLO TEMPORALE data scritto - data orale

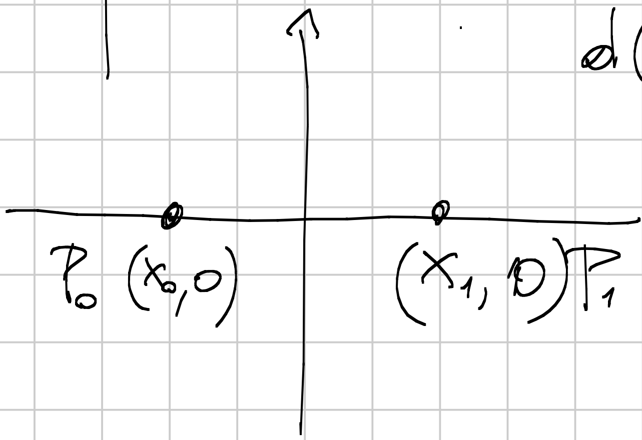
Ne segue che

DOVETE INIZIARE A STUDIARE

ORA!!!!



$$\sqrt{(x_0-x_1)^2+(y_0-y_1)^2} = d(P_0,P_1)$$



$$d(P_0,P_1) = \sqrt{(x_0-x_1)^2} = \cancel{(x_0-x_1)} = |x_0-x_1|$$

(2) $$|x| \underset{\text{Def}}{=} \begin{cases} x & x > 0 \\ -x & x \leq 0 \end{cases}$$

(1) $$|x| \underset{\text{Def}}{=} \max\{x, -x\}$$

$$|x| = \max\{x,-x\} = \begin{cases} x, & \text{se } x > 0 \\ 0, & \text{se } x = 0 \\ -x, & \text{se } x < 0 \end{cases}$$

Infatti,

$$x < 0 \Rightarrow -x > 0 > x \Rightarrow \max\{x,-x\} = -x$$

quindi se $x < 0$ allora $|x| = -x$

$x>0 \Rightarrow x>0>-x \Rightarrow \max\{x,-x\}=x$

quindi se $x>0$ allora $|x|=x$

$x=0 \Rightarrow x=-x=0 \Rightarrow \max\{x,-x\}=0$

quindi se $x=0$ allora $|x|=0$

$\forall x \in \mathbb{R}$ $|x| \underset{Def}{=} \max\{x,-x\}$ $\forall x \in \mathbb{R}$ Valore Assoluto (o Modulo) di x

(1) $|x| = \begin{cases} x, & se\ x>0 \\ -x, & se\ x \leq 0 \end{cases}$

(2) $|x|=0$ se e soltanto se, $x=0$

(3) $|x|=|-x|$ (il modulo è una funzione pari)

(4) $-|x| \leq x \leq |x|$

$\forall x,y \in \mathbb{R}$

(5) $|x| \leq y$ se, e soltanto se, $-y \leq x \leq y$

(6) $|x| \geq y$ se, e soltanto se, $[(x \geq y)$ or $(-x \geq y)]$

(7) $|x+y| \leq |x|+|y|$ (disuguaglianza Triangolare)

(8) $\big||x|-|y|\big| \leq |x-y|$

(1) È già stata provata

(2) $x=0 \Rightarrow |x|=\max\{0,-0\}=0$

"$|x|=0 \Rightarrow x=0$" equivale a (contronominale)

"$x \neq 0 \Rightarrow |x| \neq 0$"

$x \neq 0 \nearrow x>0 \Rightarrow |x|=\max\{x,-x\}=x>0 \searrow x \neq 0$

$x \neq 0 \searrow x<0 \Rightarrow |x|=\max\{x,-x\}=-x>0 \nearrow x \neq 0$

(3) $|x| = \max\{x, -x\}$ $\Longrightarrow |x| = |-x|$

$|-x| = \max\{-x, -(-x)\} = \max\{-x, x\}$

(4) $\left.\begin{array}{l} |x| = \max\{x, -x\} \geq x \\ |x| = \max\{x, -x\} \geq -x \Longleftrightarrow -|x| \leq x \end{array}\right\} \Longrightarrow -|x| \leq x \leq |x|$

(5) $|x| \leq y \Longleftrightarrow \max\{x, -x\} \leq y$

$\Longleftrightarrow x \leq y$ e $-x \leq y$

$\Longleftrightarrow x \leq y$ e $x \geq -y$

$\Longleftrightarrow -y \leq x \leq y$

(6) $|x| \geq y \Longleftrightarrow \max\{x, -x\} \geq y$

$\Longleftrightarrow [x \geq y$ o $-x \geq y]$

(7) $\begin{array}{l} -|x| \leq x \leq |x| \\ -|y| \leq y \leq |y| \end{array} \Longrightarrow -(|x| + |y|) \leq x + y \leq |x| + |y|$

per la (5) $\longrightarrow \Longrightarrow |x+y| \leq |x| + |y|$

(8) $\begin{cases} |x| = |(x-y) + y| \leq |x-y| + |y| \\ |y| = |y - x + x| \leq |y-x| + |x| \end{cases}$

$\Longleftrightarrow \begin{cases} |x| - |y| \leq |x-y| \\ -|y-x| \leq |x| - |y| \end{cases} \Longleftrightarrow -|x-y| \leq |x| - |y| \leq |x-y|$

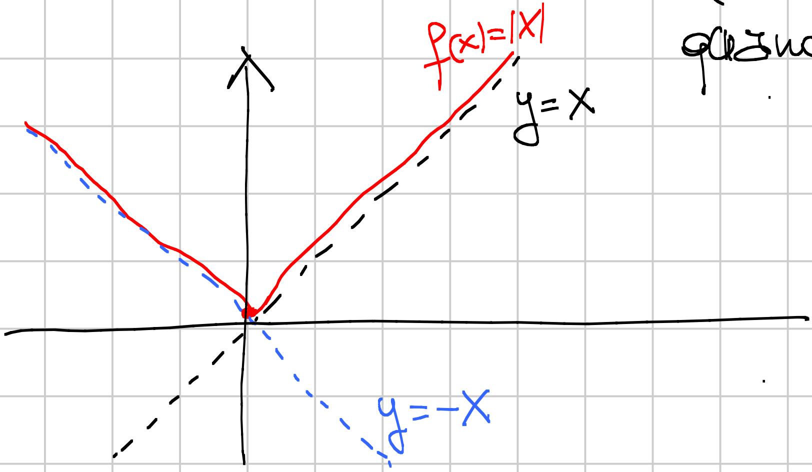
$\Updownarrow$ $\Longleftarrow$ per la (5)

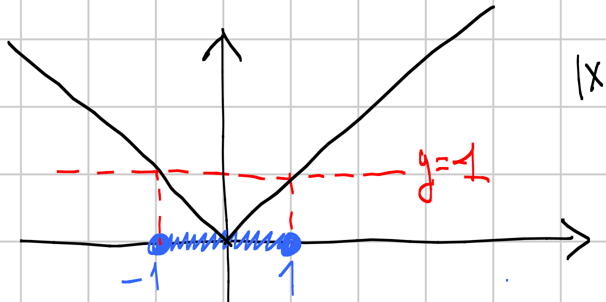
$||x| - |y|| \leq |x-y|$

Grafico di $f(x) = |x|$

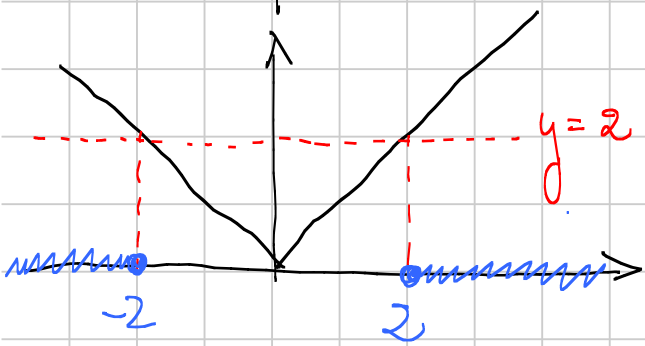
$$|x| = \begin{cases} x & \text{se } x > 0 \\ -x & \text{se } x \leq 0 \end{cases}$$ ovvero $f(x) = x$ coincide con $g(x) = |x|$ quando $x > 0$

$h(x) = -x$ coincide con $g = |x|$ quando $x \leq 0$





$|x| \leq 1 \iff x \in [-1, 1]$



$|x| \geq 2$

$\Updownarrow$

$x \in]-\infty, -2] \cup [2, +\infty[$

$$f^+(x) = \frac{|f| + f}{2} = \begin{cases} f(x) & \text{se } f(x) > 0 \\ 0 & \text{se } f(x) \leq 0 \end{cases}$$ Parte positiva di f

$$f^-(x) = \frac{|f| - f}{2} = \begin{cases} 0 & \text{se } f(x) > 0 \\ -f(x) & \text{se } f(x) \leq 0 \end{cases}$$ Parte negativa di f

Nota Bene: f può assumere segno positivo o negativo, però $\forall x \in \mathrm{dom}(f)$ $f^+(x) \geq 0$ e $f^-(x) \geq 0$

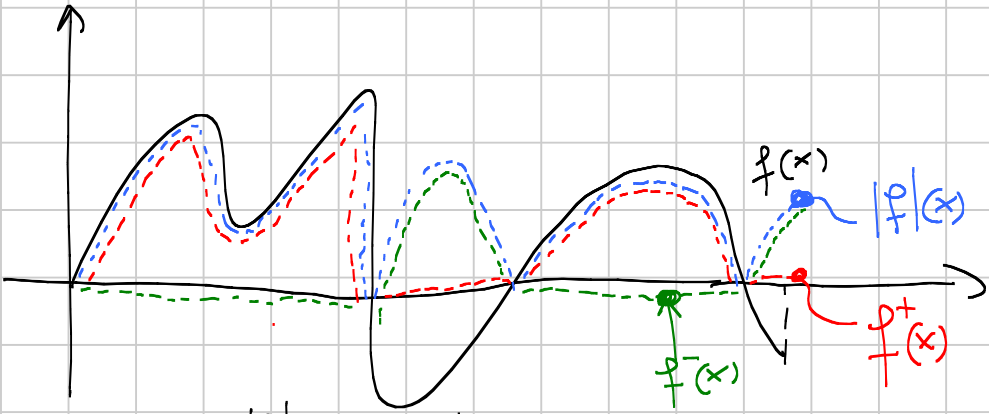
Esercizio 1.1.1 *Determinare le soluzioni di* $|x-3| = |2x-3| - 2$

Esercizio 1.1.2 *Determinare le soluzioni di* $|x-1| = |x+2| - 1$

Esercizio 1.1.3 *Determinare le soluzioni di* $|2x - |x^2-3|| < 1$

Si possono poi definire

- $f^+ = \dfrac{|f|+f}{2}$, la "parte positiva di f" $\qquad |f| \geq f$
- $f^- = \dfrac{|f|-f}{2}$, la "parte negativa di f" $\qquad |f| \geq -f$



$$f^+(x) - f^-(x) = \frac{|f|+f}{2} - \frac{|f|-f}{2} = f(x)$$

$$f^+(x) + f^-(x) = \quad // \quad + \quad // \quad = |f|(x)$$

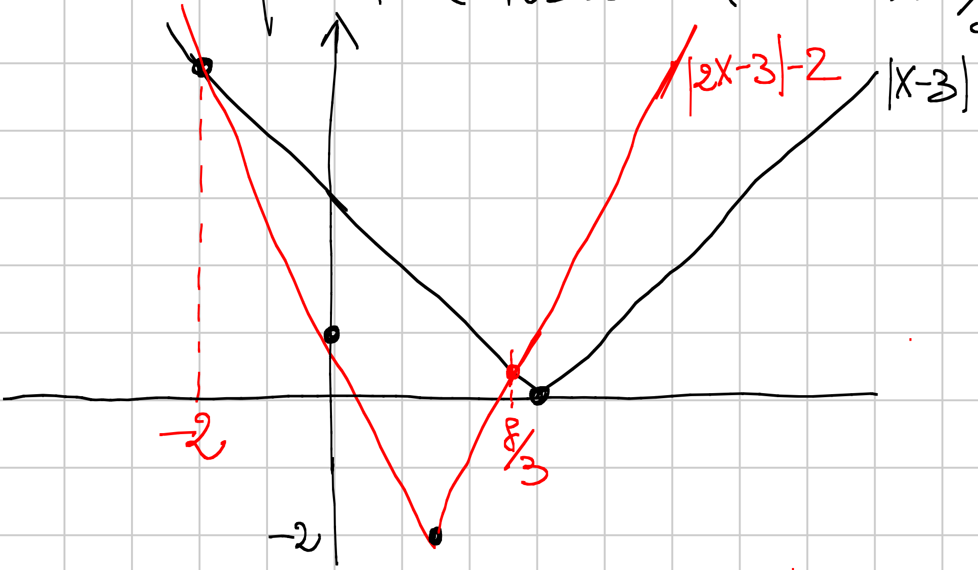
Determinare le soluzioni di

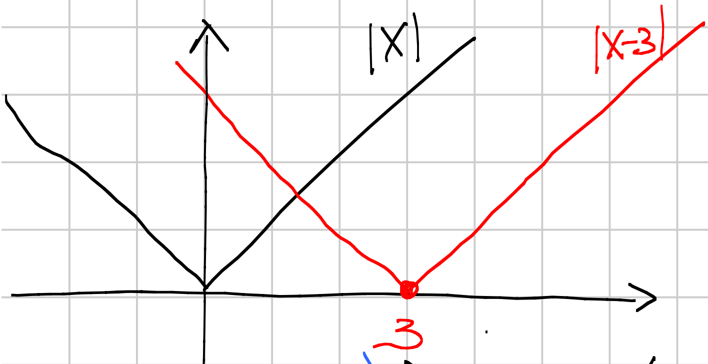
$$|x-3| = |2x-3| - 2$$

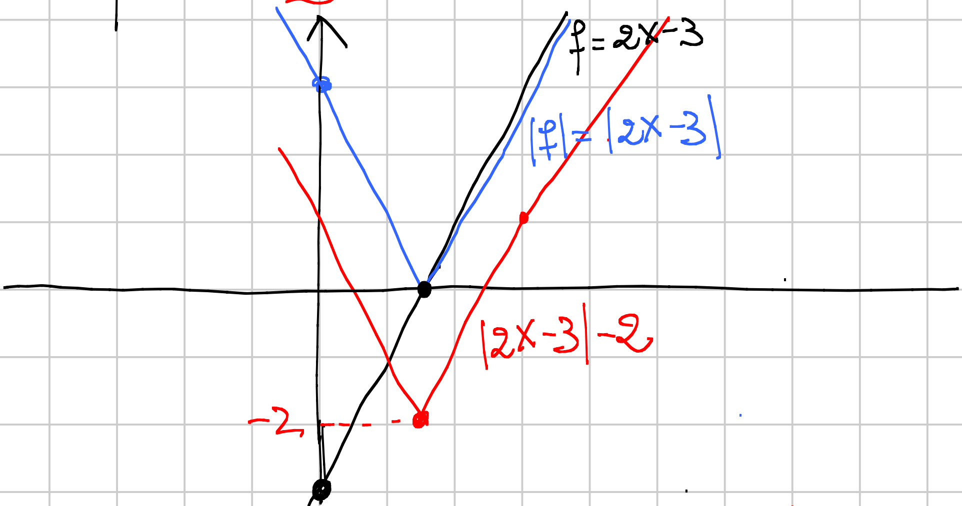
$$\begin{cases} x-3>0 \\ 2x-3>0 \\ x\cancel{-3}=2x\cancel{-3}-2 \end{cases} \quad \begin{cases} x-3>0 \\ 2x-3\leq 0 \\ x-3=3-2x-2 \end{cases} \quad \begin{cases} x-3\leq 0 \\ 2x-3>0 \\ 3-x=2x-3-2 \end{cases} \quad \begin{cases} x-3\leq 0 \\ 2x-3\leq 0 \\ \cancel{3}-x=\cancel{3}-2x-2 \end{cases}$$

$$\cancel{\begin{cases} x>3 \\ x>3/2 \\ x=2 \end{cases}} \quad \cancel{\begin{cases} x>3 \\ x\leq 3/2 \end{cases}} \text{ incompatibili} \quad \begin{cases} x\leq 3 \\ \frac{3}{2}<x \\ x=8/3 \end{cases} \quad \begin{cases} x\leq 3 \\ x\leq 3/2 \\ x=-2 \end{cases}$$

e dunque le soluzioni sono $x=8/3$ e $x=-2$







Determinare tutte le soluzioni di

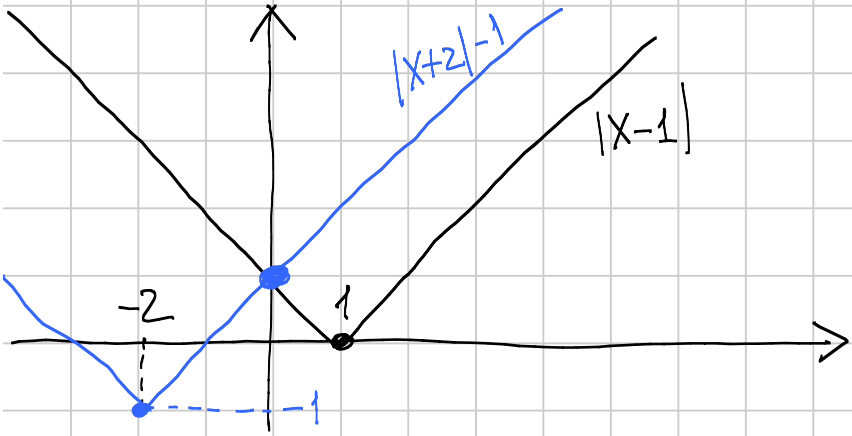
$$|x-1| = |x+2| - 1$$

$$\begin{cases} x-1>0 \\ x+2>0 \\ x-1 = x+2-1 \end{cases} \qquad \begin{cases} x-1>0 \\ x+2\le 0 \\ x-1 = -x-2-1 \end{cases} \qquad \begin{cases} x-1\le 0 \\ x+2>0 \\ 1-x = x+2-1 \end{cases} \qquad \begin{cases} x-1\le 0 \\ x+2\le 0 \\ 1-x = -x-2-1 \end{cases}$$

Impossibile impossibile impossibile

$$\begin{cases} -2<x\le 1 \\ x=0 \end{cases}$$

$x=0$ è l'unica soluzione



Determinare le soluzioni di

$$\left|2x-|x^2-3|\right|<1$$

$$\Longleftrightarrow \quad -1<2x-|x^2-3|<1$$

$$\Longleftrightarrow \begin{cases} -1<2x-|x^2-3| \\ 2x-|x^2-3|<1 \end{cases} \Longleftrightarrow \begin{cases} |x^2-3|<2x+1 \\ 2x-1<|x^2-3| \end{cases}$$

$$\Longleftrightarrow \begin{cases} -2x-1<x^2-3<2x+1 \\ 2x-1<x^2-3 \quad \text{o} \quad 2x-1<3-x^2 \end{cases}$$

$$\Longleftrightarrow \begin{cases} -2x-1<x^2-3 \\ x^2-3<2x+1 \\ 2x-1<x^2-3 \end{cases} \quad \text{o} \quad \begin{cases} -2x-1<x^2-3 \\ x^2-3<2x+1 \\ 2x-1<3-x^2 \end{cases}$$

$$\begin{cases} -2x-1 < x^2-3 \\ x^2-3 < 2x+1 \\ 2x-1 < x^2-3 \end{cases} \quad ó \quad \begin{cases} -2x-1 < x^2-3 \\ x^2-3 < 2x+1 \\ 2x-1 < 3-x^2 \end{cases}$$

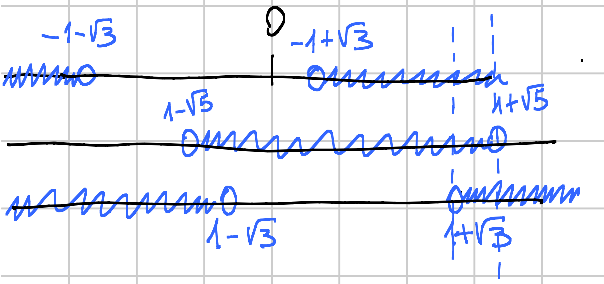
$$\begin{cases} x^2+2x-2 > 0 \\ x^2-2x-4 < 0 \\ x^2-2x-2 > 0. \end{cases} \quad ó \quad \begin{cases} x^2+2x-2 > 0 \\ x^2-2x-4 < 0 \\ x^2+2x-4 < 0 \end{cases}$$

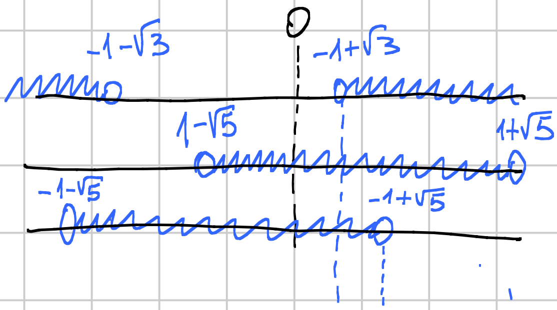
$x_{1,2} = -1 \pm \sqrt{3}$ $\qquad$ $x_{1,2} = -1 \pm \sqrt{3}$

$x_{3,4} = 1 \pm \sqrt{5}$ $\qquad$ $x_{3,4} = 1 \pm \sqrt{5}$

$x_{5,6} = 1 \pm \sqrt{3}$ $\qquad$ $x_{7,8} = -1 \pm \sqrt{5}$

$$\begin{cases} x \in]-\infty, -1-\sqrt{3}[\cup]-1+\sqrt{3}, +\infty[\\ x \in]1-\sqrt{5}, 1+\sqrt{5}[\\ x \in]-\infty, 1-\sqrt{3}[\cup]1+\sqrt{3}, +\infty[\end{cases} \quad ó \quad \begin{cases} x \in]-\infty, -1-\sqrt{3}[\cup]-1+\sqrt{3}, +\infty[\\ x \in]1-\sqrt{5}, 1+\sqrt{5}[\\ x \in]-1-\sqrt{5}, -1+\sqrt{5}[\end{cases}$$





$x \in]1+\sqrt{3}, 1+\sqrt{5}[$ $\qquad$ $x \in]-1+\sqrt{3}, -1+\sqrt{5}[$

$$\Rightarrow \quad x \in]-1+\sqrt{3}, -1+\sqrt{5}[\cup]1+\sqrt{3}, 1+\sqrt{5}[$$