Ricerca delle primitive - parte 3

Titolo nota 20/11/2012

Abbiamo calcolato $\int \frac{dx}{1+x^2} = \operatorname{arctg} x + c \quad c \in \mathbb{R}$

" " $\int \frac{dx}{(1+x^2)^2}$

$$\int \frac{dx}{(1+x^2)^2} = \int \frac{dx}{1+x^2} - \frac{1}{2}\int \frac{2x}{(x^2+1)^2} \cdot x \, dx$$

$$= \operatorname{arctg} x - \frac{1}{2}\left\{ -\frac{1}{1+x^2} \cdot x + \int \frac{dx}{1+x^2} \right\}$$

$$= \frac{1}{2} \operatorname{arctg} x + \frac{1}{2} \frac{x}{1+x^2} + c \quad c \in \mathbb{R}$$

e dunque abbiamo ricondotto il calcolo di

$\int \frac{1}{(1+x^2)^2}$ al calcolo di $\int \frac{dx}{1+x^2}$

Analogamente il calcolo di $\int \frac{1}{(1+x^2)^m} dx$

va ricondotto al calcolo di $\int \frac{dx}{(1+x^2)^{m-1}}$

Esempio Calcolare $\int \frac{dx}{(1+x^2)^3}$

$$\int \frac{1+x^2-x^2}{(1+x^2)^3}\,dx = \underbrace{\int \frac{dx}{(1+x^2)^2}}_{=\frac{1}{2}\left(\operatorname{arctg} x + \frac{x}{x^2+1}\right)} - \int \frac{x^2\,dx}{(1+x^2)^3}$$

$$= \frac{1}{2}\left(\operatorname{arctg} x + \frac{x}{x^2+1}\right) + \frac{1}{4}\int \frac{-4x}{(1+x^2)^3}\cdot x\,dx$$

$$= \frac{1}{2}\left(\operatorname{arctg} x + \frac{x}{x^2+1}\right) + \frac{1}{4}\left\{\frac{1}{(1+x^2)^2}\cdot x - \int \frac{1}{(1+x^2)^2}\right\}$$

$$= \frac{3}{8}\left(\operatorname{arctg} x + \frac{x}{x^2+1}\right) + \frac{1}{4}\,\frac{x}{(1+x^2)^2} + c \qquad c \in \mathbb{R}$$

Verifica

$$F'(x) = \frac{3}{8}\left(\frac{1}{1+x^2} + \frac{1}{1+x^2} - \frac{2x^2}{(1+x^2)^2}\right) + \frac{1}{4}\,\frac{(1+x^2)^{\cancel{2}} - 4x^2\cancel{(1+x^2)}}{(1+x^2)^{\cancel{4}\,3}}$$

$$= \frac{3}{\cancel{8}\,4}\;\frac{\cancel{2}\,1 + \cancel{2x^2} - \cancel{2x^2}}{(1+x^2)^2} + \frac{1}{4}\;\frac{1-3x^2}{(1+x^2)^3}$$

$$= \frac{1}{4}\;\frac{3(1+x^2)+1-3x^2}{(1+x^2)^3} = \frac{1}{(1+x^2)^3} \qquad \checkmark$$

Esempio Calcolare $\displaystyle\int \frac{x}{(x^2+1)^2}\,dx = \frac{1}{2}\operatorname{arctg} x - \frac{1}{2}\frac{x}{x^2+1} + C$

$C \in \mathbb{R}$

$$\frac{x^2}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$$

$$= \frac{Ax(x^2+1) + B(x^2+1) + Cx + D}{(x^2+1)^2}$$

$$Ax^3 + Bx^2 + (A+C)x + B + D = x^2$$

$$\begin{cases} A = 0 \\ B = 1 \\ A + C = 0 \\ B + D = 0 \end{cases} \qquad \begin{cases} A = 0 \\ B = 1 \\ C = 0 \\ D = -1 \end{cases}$$

$$f(x) = \frac{1}{x^2+1} - \frac{1}{(x^2+1)^2}$$

$$\int \frac{1}{(x^2+1)^2}\,dx = \int \frac{1+x^2}{(1+x^2)^2}\,dx - \int \frac{x^2}{(1+x^2)^2}$$

$$= \operatorname{arctg} x - \frac{1}{2}\int \underbrace{\frac{2x}{(1+x^2)^2}}_{u'} \cdot \underbrace{x}_{v}\,dx$$

$$= \operatorname{arctg} x - \frac{1}{2}\left\{\left(-\frac{1}{1+x^2}\right)\cdot x + \int \frac{dx}{1+x^2}\right\}$$

$$= \operatorname{arctg} x + \frac{1}{2}\,\frac{x}{x^2+1} - \frac{1}{2}\operatorname{arctg} x + c$$

$$= \frac{1}{2}\operatorname{arctg} x + \frac{1}{2}\,\frac{x}{x^2+1} + c \qquad c\in\mathbb{R}$$

$$\Downarrow$$

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \int \frac{dx}{x^2+1} - \int \frac{dx}{(x^2+1)^2}$$

$$= \operatorname{arctg} x - \frac{1}{2}\operatorname{arctg} x - \frac{1}{2}\,\frac{x}{x^2+1} + c$$

$$= \frac{1}{2}\operatorname{arctg} x - \frac{1}{2}\,\frac{x}{x^2+1} + c \qquad c\in\mathbb{R}$$

Esempio Calcolare $\int \frac{dx}{\sqrt{x}(x+1)}$

dim

$$\int \frac{dx}{\sqrt{x}(x+1)} \underset{t=\sqrt{x}}{=} \left(\int \frac{1}{t(t^2+1)} \cdot 2t\,dt \right)_{t=\sqrt{x}}$$

$x = t^2$

$dx = \frac{dx}{dt} dt = 2t\,dt$

$$= \left(\int \frac{2\,dt}{t^2+1} \right)_{t=\sqrt{x}}$$

$$= \left(2\,\mathrm{arctg}\, t + c \right)_{t=\sqrt{x}} \quad c \in \mathbb{R}$$

$$= 2\,\mathrm{arctg}(\sqrt{x}) + c \quad c \in \mathbb{R}$$

Funzioni Iperboliche

$$\operatorname{senh}(x) = \frac{e^{x}-e^{-x}}{2}$$

$$\cosh(x) = \frac{e^{x}+e^{-x}}{2} \qquad \forall x \in \mathbb{R}$$

$$\operatorname{Tgh}(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

È abbastanza semplice verificare

$$\operatorname{senh}(x) = -\operatorname{senh}(-x) \quad (\text{dispari})$$

$$\cosh(x) = \cosh(-x) \quad (\text{pari}) \qquad \forall x \in \mathbb{R}$$

$$\operatorname{Tgh}(x) = -\operatorname{Tgh}(-x) \quad (\text{dispari})$$

$$\lim_{x \to \pm\infty} \cosh(x) = +\infty$$

$$\lim_{x \to +\infty} \operatorname{senh}(x) = +\infty \qquad \lim_{x \to -\infty} \operatorname{senh}(x) = -\infty$$

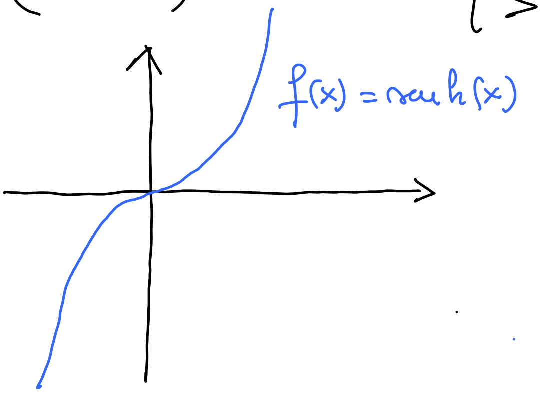
$$\lim_{x \to +\infty} \operatorname{Tgh}(x) = 1 \qquad \lim_{x \to -\infty} \operatorname{Tgh}(x) = -1$$

$$\frac{d}{dx}\left(\operatorname{senh}(x)\right) = \cosh(x) \qquad \forall x \in \mathbb{R}$$

$$\frac{d}{dx}\left(\cosh(x)\right) = \operatorname{senh}(x)$$

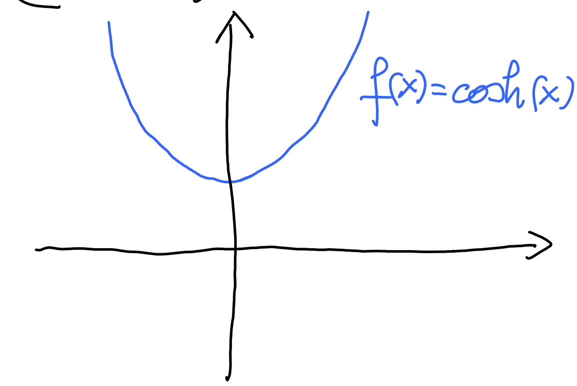
$\operatorname{senh}(0)=0, \quad (\operatorname{senh}(x))' = \cosh(x) > 0 \quad \forall x,$

$$(\operatorname{senh}(x))'' = \operatorname{senh}(x) \begin{cases} <0 & x<0 \\ >0 & x>0 \end{cases}, \quad \operatorname{senh}(x) = -\operatorname{senh}(-x)$$



$$\cosh(0)=1, \quad (\cosh(x))' = \operatorname{senh}(x) \begin{cases} <0 & x<0 \\ =0 & x=0 \\ >0 & x>0 \end{cases}$$

$$(\cosh(x))'' = \cosh(x) > 0 \quad \forall x, \quad \cosh(x) = \cosh(-x)$$



$$\cosh^2(x) - \operatorname{senh}^2(x) = \frac{e^{2x}+e^{-2x}+2}{4} - \frac{e^{2x}+e^{-2x}-2}{4} = 1$$

La funzione $f(x)=\operatorname{senh}(x)$ è strettamente crescente, dunque invertibile

$y=\operatorname{senh}(x)$ $\Leftrightarrow$ $y=\dfrac{e^x-e^{-x}}{2}$

$\Leftrightarrow$ $e^x-e^{-x}-2y=0$

$\Leftrightarrow$ $e^{2x}-2ye^x-1=0$

$\Leftrightarrow$ $e^x=y+\sqrt{y^2+1}$ (essendo $e^x>0$, va scartata la soluzione $e^x=y-\sqrt{y^2+1}$)

$\Leftrightarrow$ $x=\log\left(y+\sqrt{y^2+1}\right)$

Dunque la funzione inversa di $f(x)=\operatorname{senh}(x)$, la funzione $f^{-1}(x)=\operatorname{settsenh}(x)$ (settor seno iperbolico) ha una espressione esplicita data da

$$f^{-1}(x)=\operatorname{settsenh}(x)=\log\left(x+\sqrt{x^2+1}\right)$$

Osservazione

Se si riprende la forma trigonometrica in $\mathbb{C}$

$$z=\rho\,(\cos\theta+i\operatorname{sen}\theta)\stackrel{\cdot}{=}e^{x+i\theta}$$

dove si è posto $e^x=\rho$ $\quad e^{i\theta}=\cos\theta+i\operatorname{sen}\theta$

Ovvero si può definire $e^z=e^{\operatorname{Re}z+i\operatorname{Im}z}$

$$\stackrel{!}{=}e^{\operatorname{Re}z}\left(\cos(\operatorname{Im}z)+i\operatorname{sen}(\operatorname{Im}z)\right)$$

e questo è l'esponenziale reale esteso a tutto $\mathbb{C}$

e si ha che

$$\cos\vartheta = \frac{e^{i\vartheta}+e^{-i\vartheta}}{2} \qquad \operatorname{sen}\vartheta = \frac{e^{i\vartheta}-e^{-i\vartheta}}{2i}$$

e dunque la "parentela" tra le funzioni trigonometriche e iperboliche sono molto strette

Però vanno fatte risaltare le differenze

- $\operatorname{sen}^2\vartheta + \cos^2\vartheta = 1$
- $\operatorname{sen}\vartheta$ e $\cos\vartheta$ sono limitate
- $(\operatorname{sen}\vartheta)' = \cos\vartheta \qquad (\cos\vartheta)' = -\operatorname{sen}\vartheta$

<u>Esempio</u> Calcolare $\int \sqrt{1+x^2}\,dx$

dice

$$\int \sqrt{1+x^2}\,dx \underset{\uparrow}{=} \left(\int \cosh(t)\cdot\cosh(t)\cdot dt \right)_{x=\operatorname{senh}(t)}$$

$x = \operatorname{senh}(t)$

$dx = \frac{dx}{dt}dt = \cosh(t)dt$

$$\overset{(*)}{=} ---- = \left(\frac{1}{2}t + \frac{1}{2}\operatorname{senh}(t)\cosh(t) + c\right)_{x=\operatorname{senh}(t)} \qquad c\in\mathbb{R}$$

$$= \left(\frac{1}{2}t + \frac{1}{2}\operatorname{senh}(t)\sqrt{1+\operatorname{senh}^2(t)} + c\right)_{x=\operatorname{senh}(t)} \qquad c\in\mathbb{R}$$

$$= \frac{1}{2}\operatorname{settsenh}(x) + \frac{1}{2}x\sqrt{1+x^2} + c \qquad c\in\mathbb{R}$$

$$= \frac{1}{2}\log\left(x+\sqrt{1+x^2}\right) + \frac{1}{2}x\sqrt{1+x^2} + c \qquad c\in\mathbb{R}$$

$$\int \cosh^2(t)\,dt = \int \underset{u'}{\cosh(t)}\,\underset{v}{\cosh(t)}\,dt = \underset{u}{\operatorname{senh}(t)}\,\underset{v}{\cosh(t)} - \int \underset{u}{\operatorname{senh}(t)}\,\underset{v'}{\operatorname{senh}(t)}\,dt$$

$$\overset{!}{=} \operatorname{senh}(t)\cosh(t) - \int \left(\cosh^2(t) - 1\right) dt$$

$$\overset{!}{=} \operatorname{senh}(t)\cosh(t) + t - \int \cosh^2(t)\,dt$$

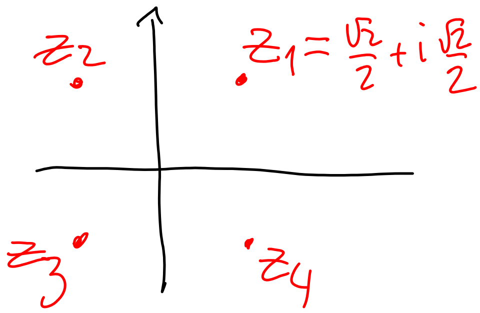
$$\Downarrow$$

$$\int \cosh^2(t) = \frac{1}{2}\left(t + \operatorname{senh}(t)\cosh(t)\right) + c \quad c \in \mathbb{R}$$

$$X^4+1=0 \qquad z_1=\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2} \qquad z_2=-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$$

$$z_3=-z_1 \qquad z_4=-z_2$$

$$z_1=\overline{z_4} \qquad z_2=\overline{z_3}$$



$$\left[(X-z_1)(X-z_4)\right]\left[(X-z_2)(X-z_3)\right]$$

$$\left[X^2-2\mathrm{Re}\,z_1+1\right]\left[X^2-2\mathrm{Re}\,z_2+1\right]$$

$$\left[X^2-\sqrt{2}X+1\right]\left[X^2+\sqrt{2}X+1\right]$$

Quindi $y = \operatorname{senh} x$ è invertibile

$y = \operatorname{senh}(x)$ $\underline{\text{sse}}$ $y = \dfrac{e^x - e^{-x}}{2}$

$\underline{\text{sse}}$ $e^x - e^{-x} - 2y = 0$

$\underline{\text{sse}}$ $e^{2x} - 2ye^x - 1 = 0$

$\underline{\text{sse}}$ $e^x = \begin{cases} \cancel{y - \sqrt{y^2+1}} & \text{è} < 0\,!! \\ y + \sqrt{y^2+1} \end{cases}$

$\underline{\text{sse}}$ $x = \log\left(y + \sqrt{y^2+1}\right)$

ovvero $y = \operatorname{settsenh}(x) = \log\left(x + \sqrt{x^2+1}\right)$

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right) = \frac{1}{\sqrt{x^2+1}}$$

$\cosh(x) > 0 \ \forall x$

Oppure ricordando che $\cosh(x) = \sqrt{1+\operatorname{senh}^2(x)}$

$$\frac{d(\operatorname{settsenh}(x))}{dx} = \frac{1}{\cosh(\operatorname{settsenh}(x))} = \frac{1}{\sqrt{x^2+1}}$$

Analogamente

$$\frac{d(\operatorname{settcosh}(x))}{dx} = \frac{1}{\operatorname{senh}(\operatorname{settcosh} x)} = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\operatorname{tgh}(x)) = \frac{(\operatorname{senh}(x))'\cosh(x) - \operatorname{senh}(x)(\cosh(x))'}{\cosh^2(x)}$$

$$= \frac{\cosh^2(x) - \operatorname{senh}^2(x)}{\cosh^2(x)} = \frac{1}{\cosh^2(x)}$$

Esempio Calcolare $\int \sqrt{1+x^2}\,dx$

$$\int \sqrt{1+x^2}\,dx \underset{x=\operatorname{senh}(t)}{=} \left(\int \cosh(t)\cdot\frac{dx}{dt}\,dt\right)_{x=\operatorname{senh}(t)}$$

$x = \operatorname{senh}(t)$

$\frac{dx}{dt} = \cosh(t)$

$$= \left(\int \cosh^2(t)\,dt\right)_{x=\operatorname{senh}(t)}$$

$$= \left(\frac{t}{2} + \frac{1}{2}\operatorname{senh}(t)\cosh(t) + c\right)_{x=\operatorname{senh}(t)}$$

$$= \frac{1}{2}\log\left(x+\sqrt{x^2+1}\right) + \frac{1}{2}x\sqrt{x^2+1}\Big) + c \quad c\in\mathbb{R}$$

$$\int \underset{u'}{\cosh(t)} \cdot \underset{v}{\cosh(t)}\, dt = \underset{u}{\operatorname{senh}(t)}\, \underset{v}{\cosh(t)} - \int \underset{u}{\operatorname{senh}(t)}\, \underset{v'}{\operatorname{senh}(t)}\, dt$$

$$= \operatorname{senh}(t)\cosh(t) - \int \left(\cosh^2(t) - 1\right) dt$$

$$\Downarrow$$

$$\int \cosh^2(t) = \frac{t}{2} + \frac{1}{2}\operatorname{senh}(t)\cosh(t) + c \qquad c \in \mathbb{R}$$

Trasformazioni razionalizzanti

$$\operatorname{sen} x = \operatorname{sen}\left(2 \cdot \frac{x}{2}\right) = \frac{2 \operatorname{sen}(x/2) \cos(x/2)}{\operatorname{sen}^2(x/2) + \cos^2(x/2)}$$

$$\operatorname{sen} x = \frac{2 \operatorname{Tg}(x/2)}{1 + \operatorname{Tg}^2(x/2)}$$

$$\cos x = \cos\left(2 \cdot x/2\right) = \frac{\cos^2(x/2) - \operatorname{sen}^2(x/2)}{\operatorname{sen}^2(x/2) + \cos^2(x/2)}$$

$$\cos x = \frac{1 - \operatorname{Tg}^2(x/2)}{1 + \operatorname{Tg}^2(x/2)}$$

Quindi, se si deve calcolare la primitiva di una funzione $f(\operatorname{sen} x, \cos x)$ conviene introdurre la nuova variabile

$$t = \operatorname{Tg}\left(\frac{x}{2}\right) \qquad x = 2 \operatorname{arcTg} t$$

che permetti di trasformare la funzione in una funzione razionale della nuova variabile t

$$\frac{dt}{dx} = \frac{1}{2} \frac{1}{\cos^2 \frac{x}{2}} = \frac{1}{2}\left(1 + \operatorname{Tg}^2\left(\frac{x}{2}\right)\right)$$

$$\frac{dx}{dt} = \frac{2}{1 + t^2}$$

Esempio Calcolare $\displaystyle\int \frac{\operatorname{sen} x}{1+\cos x}\,dx$

direi

$$\int \frac{\operatorname{sen} x}{1+\cos x}\,dx \underset{x=2\operatorname{arctg} t}{=} \left(\int \frac{2t}{1+t^2}\cdot\frac{1}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt\right)_{t=\operatorname{tg}\frac{x}{2}}$$

$$\frac{dx}{dt}=\frac{2}{1+t^2}$$

$$=\left(\int \frac{2t}{1+t^2}\cdot\frac{1}{\frac{2}{\cancel{1+t^2}}}\cdot\frac{2}{\cancel{1+t^2}}\,dt\right)_{t=\operatorname{Tg}\frac{x}{2}}$$

$$=\left(\log(1+t^2)+c\right)_{t=\operatorname{Tg}\frac{x}{2}} \qquad c\in\mathbb{R}$$

$$=\log\left(1+\operatorname{Tg}^2\left(\frac{x}{2}\right)\right)+c \qquad c\in\mathbb{R}$$

Verifica

$$\frac{d}{dx}\left(\log\left(1+\operatorname{Tg}^2\frac{x}{2}\right)\right)=\frac{1}{\cancel{1+\operatorname{Tg}^2\left(\frac{x}{2}\right)}}\cdot 2\operatorname{Tg}\frac{x}{2}\cdot\cancel{\left(1+\operatorname{Tg}^2\left(\frac{x}{2}\right)\right)}\frac{1}{2}$$

$$=\operatorname{Tg}\frac{x}{2} \qquad \text{corretto?}$$

Esempio Calcolare $\int \frac{1}{\sqrt{x}(x+1)} dx$

Poniamo $\sqrt{x}=t$ ovvero $x=t^2$ $\frac{dx}{dt}=2t$

$$\int \frac{1}{\sqrt{x}\,(x+1)} dx \underset{\sqrt{x}=t}{=} \left(\int \frac{1}{t(t^2+1)} \cdot \frac{dx}{dt} dt\right)_{t=\sqrt{x}}$$

$$= \left(\int \frac{1}{\cancel{t}(t^2+1)} \cdot 2\cancel{t}\, dt\right)_{t=\sqrt{x}} = 2\,\mathrm{arctg}\sqrt{x}+c \qquad c\in\mathbb{R}$$

Esempio Calcolare $\int \frac{dx}{1+x^4}$

dim

Le radici di $1+x^4=0$ sono date da

$x_1=\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$ $x_2=-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$ $x_3=-\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}$ $x_4=\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}$

$$(x-x_1)(x-x_4)=x^2+x(\sqrt{2})+1$$

$$(x-x_2)(x-x_3)=x^2-x\sqrt{2}+1$$

$$x^4+1=(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)$$

$$\frac{1}{x^4+1}=\frac{Ax+B}{x^2+\sqrt{2}x+1}+\frac{Cx+D}{x^2-\sqrt{2}x+1}$$

$$=\frac{Ax^3-\sqrt{2}Ax^2+Ax+Bx^2-\sqrt{2}Bx+B}{x^4+1}+\dots$$

$$\dots\frac{Cx^3+\sqrt{2}Cx^2+Cx+Dx^2+\sqrt{2}Dx+D}{}$$

$A+C=0$ $\quad -\sqrt{2}A+B+\sqrt{2}C+D=0$ $\quad A-\sqrt{2}B+C+\sqrt{2}D=0$

$B+D=1$

$$\begin{cases} A+C=0 \\ B+D=\sqrt{2}(A-C) \\ A+C=\sqrt{2}(B-D) \\ B+D=1 \end{cases} \qquad \begin{cases} A=-C \\ 1=\sqrt{2}(-2C) \\ B=D=1/2 \\ B=\frac{1}{2} \end{cases} \qquad \begin{array}{l} A=+\frac{1}{2\sqrt{2}} \\ B=1/2 \\ C=-\frac{1}{2\sqrt{2}} \\ D=1/2 \end{array}$$

$$\frac{1}{x^4+1}=\frac{1}{2\sqrt{2}}\,\frac{x+\sqrt{2}}{x^2+\sqrt{2}x+1}-\frac{1}{2\sqrt{2}}\,\frac{x-\sqrt{2}}{x^2-\sqrt{2}x+1}$$

$$\int\frac{x+\sqrt{2}}{x^2+\sqrt{2}x+1}=\frac{1}{2}\int\frac{2x+\sqrt{2}}{x^2+\sqrt{2}x+1}+\frac{1}{\sqrt{2}}\int\frac{1}{\left(x^2+2\cdot\frac{\sqrt{2}}{2}x+\frac{1}{2}\right)+\frac{1}{2}}$$

$$=\frac{1}{2}\log\left(x^2+\sqrt{2}x+1\right)+\sqrt{2}\int\frac{dx}{1+\left[\left(x+\frac{\sqrt{2}}{2}\right)\cdot\sqrt{2}\right]^2}$$

$$=\frac{1}{2}\log\left(x^2+\sqrt{2}x+1\right)+\operatorname{arctg}\left(\sqrt{2}x+1\right)+c$$

Analogamente si calcola

$$\int\frac{x-\sqrt{2}}{x^2-\sqrt{2}x+1}dx=\frac{1}{2}\int\frac{2x-\sqrt{2}}{x^2-\sqrt{2}x+1}dx-\frac{3}{\sqrt{2}}\int\frac{1}{x^2-\sqrt{2}x+\frac{1}{2}+\frac{1}{2}}dx$$

$$=\frac{1}{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{3}{\sqrt{2}}\cdot\int\frac{dx}{\left(x-\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}}$$

$$=\frac{1}{2}\log\left(x^2-\sqrt{2}x+1\right)-3\sqrt{2}\int\frac{dx}{1+\left(\sqrt{2}x-1\right)^2}$$

$$=\frac{1}{2}\log\left(x^2-\sqrt{2}x+1\right)-3\operatorname{arctg}\left(\sqrt{2}x-1\right)+c \quad c\in\mathbb{R}$$