Esercizio

Per quali x $\quad \ln(x^2-3x+4) \geq \ln(4x-6) \quad (*)$

dim

$f(y)=\ln(y)$ è strettamente crescente e dunque

$$(*) \iff \begin{cases} x^2-3x+4 \geq 4x-6 \\ x^2-3x+4 > 0 \\ 4x-6 = 4\left(x-\frac{3}{2}\right) > 0 \end{cases} \qquad x_{1,2} = \frac{3 \pm \sqrt{9-16}}{2}$$

$$\iff \begin{cases} x^2-7x+10 = (x-5)(x-2) \geq 0 \\ x \in \mathbb{R} \\ x \in \left]\frac{3}{2}, +\infty\right[\end{cases} \qquad \left(\begin{cases} (x-5)(x-2) \geq 0 \\ x \in \mathbb{R} \\ \frac{3}{2} < x \end{cases} \right)$$

$$\iff \begin{cases} x \in \left]-\infty, 2\right] \cup \left[5, +\infty\right[\\ x \in \mathbb{R} \\ x \in \left]\frac{3}{2}, +\infty\right[\end{cases}$$

$$\iff x \in \left]\frac{3}{2}, 2\right] \cup \left[5, +\infty\right[\qquad \blacksquare$$

Esercizio

Per quali valori di x $\quad 3\ln x - \frac{12}{\ln x} < 5 \quad (*)$

dim

$$(*) \iff \begin{cases} x > 0 \\ x \neq 1 \\ \frac{3(\ln x)^2 - 12 - 5\ln x}{\ln x} < 0 \end{cases}$$

$$\iff \begin{cases} x > 0 \\ x \neq 1 \\ \ln x < 0 \\ 3(\ln x)^2 - 5\ln x - 12 > 0 \end{cases} \quad \text{o} \quad \begin{cases} x > 0 \\ x \neq 1 \\ \ln x > 0 \\ 3(\ln x)^2 - 5\ln x - 12 < 0 \end{cases}$$

$$\iff \begin{cases} 0 < x < 1 \\ 3(\ln x)^2 - 5\ln x - 12 > 0 \end{cases} \quad \text{o} \quad \begin{cases} 1 < x \\ 3(\ln x)^2 - 5\ln x - 12 < 0 \end{cases}$$

$$3y^2 - 5y - 12 = 0 \qquad y_{1,2} = \frac{5 \pm \sqrt{25+144}}{6} = \frac{5 \pm 13}{6} \begin{cases} -4/3 \\ 3 \end{cases}$$

$$\Longleftrightarrow \begin{cases} 0<x<1 \\ \ln x < -\frac{4}{3} \\ \quad = \ln e^{-4/3} \end{cases} \text{ o } \cancel{\begin{cases} 0<x<1 \\ 3<\ln x \end{cases}} \text{ o } \begin{cases} 1<x \\ -\frac{4}{3} < \ln x < 3 \\ \quad \overset{\shortparallel}{\ln e^{-4/3}} \qquad \overset{\shortparallel}{\ln e^{3}} \end{cases}$$

$$\Longleftrightarrow \begin{cases} 0<x<1 \\ x<e^{-4/3} \end{cases} \text{ o } \begin{cases} 1<x \\ e^{-4/3}<x<e^3 \end{cases}$$

$$\Longleftrightarrow x \in \left]0, e^{-4/3}\right[\quad \text{o} \quad x \in \left]1, e^3\right[$$

$$\Longleftrightarrow x \in \left]0, e^{-4/3}\right[\cup \left]1, e^3\right[\qquad \blacksquare$$

Disuguaglianze Irrazionali

① $\sqrt{f(x)} > g(x)$ ② $\sqrt{f(x)} < g(x)$ ③ $\sqrt{f(x)} < \sqrt{g(x)}$

Esercizio

Per quali valori di x $\sqrt{x-1} > 2x-3$ $(*)$

dim

$$(*) \Longleftrightarrow \begin{cases} x-1 \geq 0 \\ 2x-3<0 \end{cases} \text{ o } \begin{cases} x-1 \geq 0 \\ 2x-3 \geq 0 \\ x-1 > (2x-3)^2 \end{cases} (+)$$

$\cancel{\sqrt{\ } = \pm}$ $\boxed{\sqrt{\cdots} \geq 0}$

$$\Longleftrightarrow \begin{cases} x \geq 1 \\ x < \frac{3}{2} \end{cases} \text{ o } \begin{cases} x \geq 1 \\ x \geq 3/2 \\ 4x^2-12x+9-x+1<0 \end{cases}$$

$$\Longleftrightarrow x \in \left[1, \frac{3}{2}\right[\text{ o } \begin{cases} x \geq 3/2 \\ 4x^2-13x+10 = 4\left(x-\frac{5}{4}\right)(x-2) < 0 \end{cases}$$

$$x_{1,2} = \frac{13 \pm \sqrt{169-160}}{8} \begin{cases} 5/4 \\ 2 \end{cases}$$

$$\iff x \in [1, 3/2[\quad \text{o} \quad \begin{cases} x \geq 3/2 \\ x \in]5/4, 2[\end{cases}$$

$$\iff x \in [1, 3/2[\cup [3/2, 2[= [1, 2[$$

Oss $x^2 > 3 \iff x > \sqrt{3}$ o $x < -\sqrt{3}$

mentre

$$\begin{cases} x^2 > 3 \\ x \geq 0 \end{cases} \iff x > \sqrt{3}$$

Ne segue che $\sqrt{f(x)} > g(x) \iff \begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases}$ o $\begin{cases} f(x) \geq 0 \\ g(x) \geq 0 \\ \sqrt{f(x)} > g(x) \end{cases}$

$$\iff \begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \quad \text{o} \quad \begin{cases} f(x) \geq 0 \\ g(x) \geq 0 \\ f(x) > g^2(x) \end{cases}$$

Esercizio

Per quali valori di x $\quad \sqrt{2x+1} \leq x-3 \quad (*)$

dim

$$(*) \iff \begin{cases} 2x+1 \geq 0 \\ x-3 \geq 0 \\ 2x+1 \leq (x-3)^2 \end{cases} \iff \begin{cases} x \geq -1/2 \\ x \geq 3 \\ x^2-6x+9-2x-1 \geq 0 \end{cases}$$

$$\iff \begin{cases} x \geq 3 \\ x^2-8x+8 \geq 0 \end{cases} \qquad x_{1,2} = 4 \pm \sqrt{16-8} = 4 \pm 2\sqrt{2}$$

$$\iff x \in [3,+\infty[\text{ e } x \in]-\infty, 4-2\sqrt{2}] \cup [4+2\sqrt{2}, +\infty[$$

$$\iff x \in [4+2\sqrt{2}, +\infty[$$

infatti

$$[3,+\infty[\cap \left( ]-\infty, 4-2\sqrt{2}] \cup [4+2\sqrt{2}, +\infty[\right) = [4+2\sqrt{2}, +\infty[$$

Esercizio

Determinare i valori di x tali che $\quad \sqrt{x^2-4x} > \sqrt{3-2x} \quad (*)$

dim

$$(*) \iff \begin{cases} x^2-4x \geq 0 & \text{C.E.} \\ 3-2x \geq 0 & \text{C.E.} \\ x^2-4x > 3-2x \end{cases} \iff \begin{cases} x(x-4) \geq 0 \\ \frac{3}{2} \geq x \\ x^2-2x-3 = (x-3)(x+1) > 0 \end{cases}$$

$$\iff \begin{cases} x \in]-\infty, 0] \cup [4,+\infty[\\ x \in]-\infty, \frac{3}{2}] \\ x \in]-\infty,-1[\cup]3,+\infty[\end{cases}$$

$$\iff x \in \left\{]-\infty, 0] \cup [4,+\infty[\cap]-\infty, \tfrac{3}{2}] \right\} \cap]-\infty,-1[\cup]3,+\infty[$$

$$\iff x \in]-\infty,0] \cap \left( ]-\infty,-1[\cup]3,+\infty[\right)$$

$$\iff x \in \left( ]-\infty,0] \cap]-\infty,-1[\right) \cup \left( ]-\infty,0] \cap]3,+\infty[\right)$$

$$\iff x \in]-\infty,-1[\cup \emptyset$$

Funzioni elementari

Polinomi $\qquad a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = P_n(x)$

$n \in \mathbb{N}_0$

funzioni Trigonometriche $\qquad \operatorname{sen} x$ e $\cos x$

funzioni esponenziali $\qquad a^x$ con $a > 0$

funzione modulo $\qquad |x|$

Somma, prodotto e composizione di f.ni elementari restituiscono l'insieme di Tutte le funzioni (Tenendo conto anche delle f.ni inverse)

Funzione Modulo

$$\boxed{|x| := \max\{x,-x\}}$$

Esempio

$x = 0 \qquad |0| = \max\{0,-0\} = 0$

$x = 3 \qquad |3| = \max\{3,-3\} = 3$

$x=-2$ $\quad |-2| = \max\{-2, -(-2)\} = \max\{-2, 2\} = 2$

$$|2| = \max\{2, -2\} = 2$$

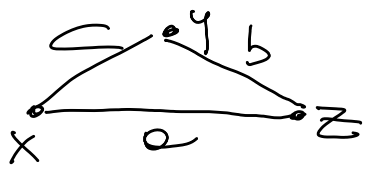
la f.ne modulo permette di introdurre una "distanza" tra punti della retta reale

Def Una f.ne $d : X \times X \longrightarrow [0, +\infty[$ si dice "distanza" tra i punti di X se

1) $d(x,y) \geq 0$; $d(x,y) = 0$ se $x = y$ $\quad \forall x, y \in X$

2) $d(x,y) = d(y,x)$ $\quad \forall x, y \in X$

3) $d(x,y) \leq d(x,z) + d(z,y)$ $\quad \forall x, y, z \in X$



$c \leq a + b$

Pb: posto $a = 12$ $\quad b = 7$ $\quad c = 4$, esiste un triangolo con questi lati?

Proprietà $|x|$

1) $|x| = \begin{cases} x & \text{se } x > 0 \\ -x & \text{se } x \leq 0 \end{cases}$

proviamo che $\max\{x, -x\} = \begin{cases} x & \text{se } x > 0 \\ -x & \text{se } x \leq 0 \end{cases}$

se $x > 0$ allora $\max\{x, -x\} = |x| = x$

se $x = 0$ allora $\max\{0, -0\} = |0| = 0$

se $x<0$ allora $\max\{x,-x\} = |x| = -x$

e dunque la 1) è vera

ancora da definire

2) $|x| \geq 0$; $|x|=0$ se $x=0$ $\quad \forall x \in \mathbb{R}$

$$\left.\begin{aligned} &x>0 \Rightarrow |x| = \max\{x,-x\} = x > 0 \\ &x=0 \Rightarrow |x| = |0| = 0 \\ &x<0 \Rightarrow |x| = \max\{x,-x\} = -x > 0 \end{aligned}\right\} \text{dunque } |x| \geq 0$$

se $x=0$ allora $|x|=|0|=0$

resta da provare $|x|=0 \Rightarrow x=0$

per assurdo $x \neq 0 \rightarrow x>0 \Rightarrow |x| = \max\{x,-x\} = x > 0$ Assurdo

$x<0 \Rightarrow |x| =$ " $= -x > 0$ "

dunque $x=0$

3) $|x| = |-x| \quad \forall x \in \mathbb{R}$

$$\left.\begin{aligned} &|x| = \max\{x,-x\} \\ &|-x| = \max\{-x,-(-x)\} = \max\{-x,x\} \end{aligned}\right\} \text{e dunque è vera}$$

4) $-|x| \underset{(i)}{\leq} x \underset{(ii)}{\leq} |x| \quad \forall x \in \mathbb{R}$

(ii) $|x| = \max\{x,-x\} \geq x$

(i) $|x| = \max\{x,-x\} \geq -x \Rightarrow -|x| \leq x$

5) $|x| \leq y \Leftrightarrow -y \leq x \leq y \quad \forall x,y \in \mathbb{R}$

$|x| = \max\{x,-x\} \leq y \Leftrightarrow x \leq y$ e $-x \leq y$

$\Leftrightarrow x \leq y$ e $x \geq -y \Rightarrow -y \leq x \leq y$

6) $|x| \geq y \iff x \geq y$ o $-x \geq y \quad \forall x,y \in \mathbb{R}$

$|x| = \max\{x,-x\} \geq y \iff x \geq y$ o $-x \geq y$

7) $|x+y| \leq |x| + |y| \quad \forall x,y \in \mathbb{R}$

Utilizzo 2 volte la 5) infatti

$$\begin{aligned} -|x| \leq x \leq |x| \\ -|y| \leq y \leq |y| \end{aligned} \implies -(|x|+|y|) \leq x+y \leq |x|+|y|$$

$$\Updownarrow \; 4)$$

$$|x+y| \leq |x|+|y|$$

8) $\big||x|-|y|\big| \leq |x-y| \quad \forall x,y \in \mathbb{R}$

$$|x| = |(x-y)+y| \overset{7)}{\leq} |x-y|+|y| \implies \underline{|x|-|y| \leq |x-y|}$$

$$|y| = |(y-x)+x| \leq |y-x|+|x| \implies |y|-|x| \leq |x-y| \implies \underline{|x|-|y| \geq -|x-y|}$$

$$\implies -|x-y| \leq |x|-|y| \leq |x-y| \overset{4)}{\implies} \big||x|-|y|\big| \leq |x-y|$$