Abstracts of papers by Alessandro Zaccagnini

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This file contains full references and abstracts of my papers. Chronologically, these deal with the following topics:

- Large gaps between primes in arithmetic progressions [1].
- An additive problem of Hardy and Littlewood and its variants [2, 3, 4].
- The Selberg integral [5, 6].
- An elementary talk on Goldbach's problem [7].
- An "inverse" problem related to the Selberg integral [8].
- Another additive problem of the Hardy–Littlewood type [9].
- An elementary talk on the continued fraction for $\sqrt{2}$ [10].
- An elementary talk on properties of prime numbers and their relations to cryptography [11].
- A review paper on the Selberg integral and related topics [12].
- An elementary paper that deals with various approximations to π [13].
- A paper on the Mertens product over primes in arithmetic progressions [14].

- A paper dealing with an additive problem involving primes and powers of 2 [15].
- An elementary paper on the use of hand-held non-programmable calculators [16].
- A paper on an average result concerning Hardy–Littlewood numbers [17].
- A paper on the averages of the error term of the Mertens product over primes in arithmetic progressions [18].
- An elementary paper on the computation of areas without calculus [19].
- A paper on the numerical computation of the constants appearing in the Mertens product over primes in arithmetic progressions [20].
- A paper on protocols in Peer-to-Peer networks [21].
- A paper on identities satisfied by the constants appearing in the Mertens product over primes in arithmetic progressions [22].
- A paper on extremely small gaps between prime numbers, and short intervals that do not contain any prime [23].
- A paper on the numerical computation of the Mertens and Meissel–Mertens constants for primes in arithmetic progressions [24].
- An elementary paper on the Euclidean Algorithm, its relation to the Fibonacci sequence and continued fractions [25].
- A paper on a Diophantine problem with primes and powers of 2 [26].
- A paper on the Montgomery-Hooley inequality in short intervals [27].
- A paper on the asymptotic number of representations of an even integer as a sum two primes [28], and one on the number of representations of an integer as a sum of several primes [29].
- A paper on a quantitative relation between the asymptotic behaviour of the mean-square of primes in short intervals and of the pair-correlation function of the zeros of the Riemann zeta function [30].

- Two papers on Diophantine problems with powers of prime numbers: [31] and [32].
- A paper on weighted averages of the number of representations of an integer as a sum of a prime and a square [33].
- A paper on a Diophantine problem with "mixed" powers of prime numbers: [34].
- A paper on weighted averages of the number of representations of an integer as a sum of two primes [35].
- A paper on explicit relations between primes in short intervals and exponential sums over primes integer as a sum of two primes [36].
- An elementary paper on the critical reassessment of basic operations, the corresponding algorithms and their complexity [37].
- A paper on the chronological development of the studies concerning the distribution of prime numbers, from Euclid to the present [38].
- Two papers on a new, wide-ranging conjecture concerning the pair-correlation of the zeros of the Riemann ζ-function, and their applications to the distribution of prime numbers; [39] and [40].
- A paper on existence of sums of a prime and two squares of primes in very short intervals [41].
- Two papers on some asymptotic formulae for the average number of representations of an integer as the sum of a prime and a square, or sum of two prime squares, respectively, in short intervals [42], [43].
- An elementary paper on "mechanical" methods for producing prime numbers [44].
- A survey of recent results on primes in short intervals [45].
- A paper on a short-interval version of an explicit formula for the weighted number of Goldbach representations of an integer [46].
- A paper on a Diophantine problem with prime numbers [47].

- Number of representations [48]
- Cesàro average [49]

The following “papers” only appear on the web.

- These “papers” deal with properties of prime numbers: [50], [51], [52].
- Properties of prime numbers and their relation to cryptography [53].
- Properties of prime numbers: in particular, small and large gaps between consecutive primes [54].
- An account of the ideas leading to the recent developments in the “twin-prime” problem [55].
- An account of Terence Tao’s proof of the Erdős discrepancy conjecture [56].
- A simple explanation of the Riemann Hypothesis and its consequences [57].
- A account of a recent result of Lemke Oliver & Soundararajan [58].
- The text of a conference for selected high-school students [59].
- A very simple, introductory paper, on Number Theory, its main problems and modern applications [60].
- A comment concerning a new approach to the proof of the Riemann Hypothesis [61].

Papers

1 Abstract of paper [1]

Let

$$G(x;q,a) := \max_{\substack{p_n \le x \\ p_n \equiv p_{n+1} \equiv a \mod q}} (p_{n+1} - p_n),$$

where $(a,q) = 1$ and p_n, p_{n+1} are consecutive primes in the arithmetic progression $a \mod q$. We prove the following result: let C be any fixed positive number, $C < 1$. Then, uniformly for

$$1 \le q \le \exp\exp\left\{C \log\log x \frac{\log\log\log\log x}{\log\log\log x}\right\}$$

we have:

$$G(x;q,a) \geq \left(e^{\gamma}+o_C(1)\right)\phi(q)\log x \log\log x \frac{\log\log\log\log x}{(\log\log\log x)^2},$$

where γ is Euler's constant. The o-symbol may depend on C.

2 Abstract of paper [2]

Let $k \geq 2$ be an integer, and set $E_k(X) := |\{n \leq X\colon\ n \neq m^k,\ n \text{ is not a sum of a prime and a } k\text{-th power}\}|$. We prove that there exists $\delta = \delta(k) > 0$ such that $E_k(X) \ll_k X^{1-\delta}$, by means of a suitable application of the circle method, essentially a variant of Montgomery & Vaughan's method (Acta Arithmetica 1975). The proof is similar to the one given by Brünner, Perelli & Pintz (Acta Math. Hungarica 1989) in the case $k = 2$, the main new difficulty being in the treatment of the singular series.

3 Abstract of paper [3]

Let $k \geq 2$ be a fixed integer and p denote a prime. For any $n \in \mathbb{N}$ such that the polynomial $x^k - n$ is irreducible over $\mathbb{Q}$ let

$$R_k(n) := \sum_{h+m^k=n} \Lambda(h), \qquad \rho_k(n,p) := |\{m \mod p\colon\ m^k \equiv n \mod p\}|$$

and

$$\mathfrak{S}_k(n) := \prod_p \left(1 - \frac{\rho_k(n,p)-1}{p-1}\right) = \prod_p \left(1 - \frac{\rho_k(n,p)}{p}\right) \Big/ \left(1 - \frac{1}{p}\right).$$

One expects that for the integers n that we are considering one has $R_k(n) \sim n^{1/k}\mathfrak{S}_k(n)$ as $n \to \infty$. Let $L := \log N$,

$$R_k^*(n) := \sum_{\substack{h+m^k=n \\ N-Y\leq h\leq N \\ Y/2\leq m^k\leq 3/2Y}} \Lambda(h), \qquad \text{and} \qquad P_k^*(n) := \frac{1}{k} \sum_{\substack{h+m=n \\ N-Y\leq h\leq N \\ Y/2\leq m\leq 3/2Y}} m^{\frac{1}{k}-1}.$$

Building on previous work by Perelli & Pintz in the case $k = 2$, we prove

Theorem 1 *Let $k \geq 3$, ε, $A > 0$, $N^{7/12+\varepsilon} \leq Y \leq N$ and* $\max\left(Y^{1-1/k+\varepsilon},\ N^{1/2+\varepsilon}\right) \leq H \leq Y$. *Then*

$$\sum_{N \leq n \leq N+H}^{\star} |R_k^*(n) - P_k^*(n)\mathfrak{S}_k(n)|^2 \ll_{\varepsilon,A,k} HY^{2/k}L^{-A},$$

where the $\star$ means that the sum is over $n \in \mathbb{N}$ such that $x^k - n$ is irreducible over $\mathbb{Q}$.

Theorem 2 *Assume the Generalized Riemann Hypothesis and let $k \geq 2$, $\varepsilon > 0$. Then*

$$E_k(N) := |\{N \leq n \leq 2N\colon\ n \neq p + m^k\}| \ll_{k,\varepsilon} N^{1+\varepsilon-2/(kK)}$$

where $K := 2^{k-1}$.

Theorem 2 is obtained by means of Hardy & Littlewood's circle method, and Weyl's inequality, by a suitable treatment of the singular series.

4 Abstract of paper [4]

In their third paper of the "Partitio Numerorum" series, Hardy & Littlewood conjectured that for $k = 2$, 3 and for $n \to \infty$, $n \neq m^k$, there is an asymptotic formula for $r_k(n) := |\{(m,p)\colon\ n = m^k + p,\ p \text{ is a prime}\}|$ and that, in particular, $r_k(n) \to \infty$ if $n \to \infty$, $n \neq m^k$. For $k \geq 2$ let $E_k(X) := |\{n \leq X\colon\ r_k(n) = 0\}|$ be the number of exceptions to the "weak" conjecture $r_k(n) \geq 1$ for $n \geq n_0(k)$, n not a power. We prove that there exists $\delta = \delta(k) > 0$ such that $E_k(X) \ll X^{1-\delta}$, that $E_k(X+H) - E_k(X) \ll H(\log X)^{-A}$ for $X^{\frac{7}{12}(1-1/k)+\varepsilon} \leq H \leq X$, and also give estimates for the number of integers for which the asymptotic formula actually holds. Furthermore, we give explicit estimates for $\delta(k)$ under the Generalized Riemann Hypothesis. This paper contains in part [2, 3] and a sketch of the circle method, as used in these problems.

5 Abstract of paper [5]

We give an alternative proof of the well-known estimate $J(X,h) = o\big(Xh^2\big)$ for the Selberg integral

$$J(X,h) := \int_X^{2X} \big|\psi(t) - \psi(t-h) - h\big|^2 \,\mathrm{d}t$$

when $h \geq X^{1/6+\varepsilon}$, by means of Heath-Brown's identity.

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6 Abstract of paper [6]

We prove the estimate $J(x,h) = o\big(xh^2(\log x)^{-2}\big)$ for the Selberg integral

$$J(x,h) := \int_x^{2x} \left|\pi(t) - \pi(t-h) - \frac{h}{\log t}\right|^2 \mathrm{d}t,$$

when $h \geq x^{1/6-\varepsilon(x)}$, provided that $\varepsilon(x) \to 0$ as $x \to +\infty$. The proof depends on an identity of Linnik and Heath-Brown which yields a suitable Dirichlet series decomposition for the quantity that we want to estimate. This is in a form that can be attacked by means of mean value theorems for Dirichlet series.

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7 Sunto dell'articolo [7]

Si tratta del testo di una conferenza divulgativa tenuta in occasione dell'allestimento della Mostra "Oltre il Compasso" a Parma fra il 1°.10 ed il 5.11.1998. Si vuole mostrare come argomentazioni essenzialmente elementari derivate in ultima analisi dal Crivello di Eratostene permettono di "indovinare" la formula asintotica che si congettura corretta per il numero di rappresentazioni di un intero pari grande come somma di due numeri primi. La stessa tecnica è applicata a problemi analoghi. In coda si danno riferimenti a letture ed approfondimenti ulteriori.

We give essentially elementary arguments based on Eratosthenes' sieve to find the expected asymptotic formula for the number of representations of a large even number n as a sum of two primes and apply the same method to similar additive problems involving prime numbers. The argument is used to explain the irregularities observed in the number of representations, as n grows to infinity.

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8 Abstract of paper [8]

We find zero-free regions and density theorems for the Riemann zeta-function depending on non-trivial estimates for

$$J(x,\theta) := \int_x^{2x} |\psi(t) - \psi(t-\theta t) - \theta t|^2 \mathrm{d}t,$$

uniformly in some range of θ. A Corollary of our main result is the following: Assume that $J(x,\theta) = O\left(\frac{x^3\theta^2}{F(x\theta)}\right)$, uniformly for $x^{-\beta} \le \theta \le 1$, where F is a positive, increasing function which is unbounded as $x \to +\infty$, such that $F(x) = O(x^\varepsilon)$ for every $\varepsilon > 0$, and that $\beta \in (0,1)$. Then the Riemann zeta-function does not vanish in the region

$$\sigma > 1 - C\frac{\log F(t)}{\log t},$$

where C is some absolute, positive constant. The proof is based on Turán's power sum method.

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9 Abstract of paper [9]

This paper contains the proof that "almost all" positive integers can be represented as a sum of a prime and a value of a polynomial $f \in \mathbb{Z}[x]$, provided that f satisfies a necessary arithmetical condition.

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10 Sunto dell'articolo [10]

Si tratta del testo di una conferenza divulgativa che prende a pretesto il fatto che il rapporto fra le dimensioni dei lati di un foglio di carta nel formato A4 è uno dei convergenti della frazione continua di $\sqrt{2}$ per parlare di ricorrenze, geometria, il metodo di Newton ed altro, usando solo argomentazioni elementari. In coda, si danno spunti per letture ed approfondimenti ulteriori.

We take as our text the fact that the ratio of the sides of a sheet of paper in the common A4 format is one of the convergents of the continued fraction for $\sqrt{2}$ for a journey through geometry, recurrences, Newton's method, ..., using essentially only elementary arguments.

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11 Sunto dell'articolo [11]

Si studiano alcune proprietà elementari dei numeri primi e se ne discute la rilevanza alle applicazioni crittografiche.

We study some elementary properties of prime numbers and discuss their relevance to applications in modern cryptography.

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12 Abstract of paper [12]

This is a paper containing a review of results that appeared in [6] and [8].

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13 Sunto dell'articolo [13]

In questo articolo elementare si danno due dimostrazioni della formula di Archimede–Viète per π e le dimostrazioni delle formule usate da Machin, Shanks ed altri per calcolare π con centinaia di cifre a partire dal Settecento.

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14 Abstract of paper [14]

We study the Mertens product over primes in arithmetic progressions, and find a uniform version of previous results on the asymptotic formula, improving at the same time the size of the error term, and giving an alternative, simpler value for the constant appearing in the main term.

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15 Abstract of paper [15]

We prove that, for any fixed positive integer k, a suitable asymptotic formula for the number of representations of an even integer $N \in [1,X]$ as the sum of two primes and k powers of 2 holds with at most $O_k\left(X^{3/5}(\log X)^{10}\right)$ exceptions.

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16 Sunto dell’articolo [16]

In questo articolo elementare, rivolto agli insegnanti delle Scuole Medie inferiori e superiori, si tratta dell’uso della calcolatrice tascabile non programmabile, delle sue limitazioni intrinseche e delle sue potenzialità, per favorirne un uso consapevole da parte degli studenti.

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17 Abstract of paper [17]

Here we give an average result on the main term for the asymptotic formula of the Hardy–Littlewood numbers in short intervals.

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18 Abstract of paper [18]

We give estimates for the error term of the Mertens product over primes in arithmetic progressions of the Bombieri–Vinogradov and Barban–Davenport–Halberstam type.

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19 Sunto dell’articolo [19]

In questo articolo elementare, rivolto agli insegnanti delle Scuole Medie inferiori e superiori, si calcolano i valori delle aree sottese dal grafico di alcune funzioni elementari senza fare ricorso al Teorema fondamentale del calcolo integrale, usando opportune decomposizioni del dominio, e qualche identità algebrica. L’articolo contiene le dimostrazioni di tutte le identità e dei limiti notevoli utilizzati.

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20 Abstract of paper [20]

This paper reports on the explicit numerical computation with at least 100 decimal digits of the constants appearing in the Mertens product over primes in arithmetic progressions, for progressions with moduli up to 100.

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21 Abstract of paper [21]

This paper suggests an alternative protocol for the management of reputation in Peer-to-Peer networks based on DHT (Distributed Hash Tables).

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22 Abstract of paper [22]

We give new identities for the constants in the Mertens product over primes in the arithmetic progressions $a \bmod q$, extending previous work by Uchiyama, Grosswald, Williams and Moree.

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23 Abstract of paper [23]

We give a new estimate for the integral moments of primes in short intervals of the type $(p, p+h]$, and prove that for every $\lambda > 1/2$ there exists a positive proportion of primes $p \le X$ such that the interval $(p, p+\lambda \log X]$ contains at least a prime number. We improve Cheer and Goldston's result (1987) on the size of real numbers $\lambda > 1$ with the property that there is a positive proportion of integers $m \le X$ such that the interval $(m, m+\lambda \log X]$ contains no primes.

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24 Abstract of paper [24]

We give explicit numerical values with 100 decimal digits for the Mertens constant involved in the asymptotic formula for $\sum_{\substack{p \le x \\ p \equiv a \bmod q}} 1/p$ and, as a by-product, for the Meissel-Mertens constant defined as $\sum_{p \equiv a \bmod q} \big(\log(1-1/p) + 1/p\big)$, for $q \in \{3, \ldots, 100\}$ and $(q,a) = 1$.

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25 Sunto dell'articolo [25]

Partendo dal semplice problema concreto del calcolo efficiente del massimo comun divisore fra due interi positivi, si descrive l'Algoritmo di Euclide e se ne fa un'analisi di complessità parziale, scoprendo che il numero di iterazioni necessarie è massimo se gli interi dati sono termini consecutivi della successione dei numeri di Fibonacci. Si interpreta il calcolo come la frazione continua del rapporto fra gli interi dati e si generalizza alle frazioni continue infinite, concludendo con la scoperta che le frazioni continue periodiche hanno valore irrazionale quadratico."

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26 Abstract of paper [26]

We refine a recent result of Parsell (2003) on the values of the form $\lambda_1 p_1 + \lambda_2 p_2 + \mu_1 2^{m_1} + \cdots + \mu_s 2^{m_s}$, where p_1, p_2 are prime numbers, $m_1, \ldots, m_s$ are positive integers, λ_1/λ_2 is negative and irrational and λ_1/μ_1, $\lambda_2/\mu_2 \in \mathbb{Q}$.

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27 Abstract of paper [27]

We study a short-interval version of a result due to Montgomery and Hooley. Write

$$S(x,h,Q) = \sum_{q \le Q} \sum_{\substack{a=1 \\ (a,q)=1}}^{q} \left| \psi(x+h;q,a) - \psi(x;q,a) - \frac{h}{\varphi(q)} \right|^2$$

and $\kappa = 1 + \gamma + \log 2\pi + \sum_p (\log p)/p(p-1)$. Denote the expected main term by $M(x,h,Q) = hQ\log(xQ/h) + (x+h)Q\log(1+h/x) - \kappa hQ$. Let ε, $A > 0$ be arbitrary, $x^{7/12+\varepsilon} \le h \le x$ and $Q \le h$. There exists a positive constant c_1 such that

$$S(x,h,Q) - M(X,h,Q) \ll h^{1/2} Q^{3/2} \exp\left(-c_1 \frac{(\log 2h/Q)^{3/5}}{(\log\log 3h/Q)^{1/5}} \right) + h^2 \log^{-A} x.$$

Now assume *GRH* and let $\varepsilon > 0$, $x^{1/2+\varepsilon} \le h \le x$ and $Q \le h$. There exists a positive constant c_2 such that

$$S(x,h,Q) - M(x,h,Q) \ll \left(\frac{h}{Q}\right)^{1/4+\varepsilon} Q^2 + hx^{1/2} \log^{c_2} x.$$

28 Abstract of paper [28]

Let Λ be the von Mangoldt function and $R(n) = \sum_{h+k=n} \Lambda(h)\Lambda(k)$ be the counting function for the Goldbach numbers. Let $N \geq 2$ and assume that the Riemann Hypothesis holds. We prove that

$$\sum_{n=1}^{N} R(n) = \frac{N^2}{2} - 2\sum_{\rho} \frac{N^{\rho+1}}{\rho(\rho+1)} + O\big(N(\log N)^3\big),$$

where $\rho = 1/2 + i\gamma$ runs over the non-trivial zeros of the Riemann zeta-function $\zeta(s)$.

29 Abstract of paper [29]

Assuming that the Generalized Riemann Hypothesis holds, we prove an explicit formula for the number of representations of an integer as a sum of $k \geq 5$ primes. Our error terms improve by some logarithmic factors an analogous result by Friedlander & Goldston (1997).

30 Abstract of paper [30]

In this paper we obtain a quantitative version of the well-known theorem by D. A. Goldston and H. L. Montgomery on the equivalence between the asymptotic behaviours of the mean-square of primes in short intervals and of the pair-correlation function of the zeros of the Riemann zeta-function.

31 Abstract of paper [31]

We prove that if λ_1, λ_2, λ_3 and λ_4 are non-zero real numbers, not all of the same sign, λ_1/λ_2 is irrational, and α is any real number then, for any $\varepsilon > 0$ the inequality $\left|\lambda_1 p_1 + \lambda_2 p_2^2 + \lambda_3 p_3^2 + \lambda_4 p_4^2 - \alpha\right| \leq \left(\max_j p_j\right)^{-1/18+\varepsilon}$ has infinitely many solutions in prime variables $p_1, \dots, p_4$.

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32 Abstract of paper [32]

We study the distribution of the values of the form $\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3^k$, where λ_1, λ_2 and λ_3 are non-zero real numbers not all of the same sign, with λ_1/λ_2 irrational, and p_1, p_2 and p_3 are prime numbers. We prove that, when $1 < k < 4/3$, these value approximate rather closely any prescribed real number.

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33 Abstract of paper [33]

We study averages of the quantity $R_{HL}(n) = \sum_{m_1+m_2^2=n} \Lambda(m_1)$. In particular, for any $k > 1$ we give an "explicit formula" for

$$\sum_{n \leq N} R_{HL}(n) \frac{(1-n/N)^k}{\Gamma(k+1)}$$

in terms of the Gamma function evaluated at suitable combinations of the Riemann zeta-function, and of Bessel functions of complex order.

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34 Abstract of paper [34]

Let $1 < k < 33/29$. We prove that if λ_1, λ_2 and λ_3 are non-zero real numbers, not all of the same sign and such that λ_1/λ_2 is irrational, and γ is any real number, then for any $\varepsilon > 0$ the inequality $\left|\lambda_1 p_1 + \lambda_2 p_2^2 + \lambda_3 p_3^k - \gamma\right| \leq \left(\max_j p_j\right)^{-(33-29k)/(72k)+\varepsilon}$ has infinitely many solutions in prime variables p_1, p_2, p_3.

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35 Abstract of paper [35]

Let Λ be the von Mangoldt function and $R(n) = \sum_{m_1+m_2=n} \Lambda(m_1)\Lambda(m_2)$ be the weighted counting function for the Goldbach numbers. We prove that for $N \to +\infty$ we have

$$\sum_{n\le N} R(n)\frac{(1-n/N)^k}{\Gamma(k+1)} = \frac{N^2}{\Gamma(k+3)} - 2\sum_{\rho}\frac{\Gamma(\rho)}{\Gamma(\rho+k+2)}N^{\rho+1} + \sum_{\rho_1}\sum_{\rho_2}\frac{\Gamma(\rho_1)\Gamma(\rho_2)}{\Gamma(\rho_1+\rho_2+k+1)}N^{\rho_1+\rho_2} + O_k\Big(N^{1/2}\Big),$$

for $k > 1$, where ρ, with or without subscripts, runs over the non-trivial zeros of the Riemann zeta-function $\zeta(s)$.

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36 Abstract of paper [36]

Under the assumption of the Riemann Hypothesis, we prove explicit quantitative relations between hypothetical error terms in the asymptotic formulae for truncated mean-square average of exponential sums over primes and in the mean-square of primes in short intervals. We also remark that such relations are connected with a more precise form of Montgomery's pair-correlation conjecture.

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37 Sunto dell'articolo [37]

Questo è un articolo divulgativo che raccoglie il materiale didattico realizzato nell'ambito di un Laboratorio PLS a proposito delle operazioni elementari, gli algoritmi relativi, la loro complessità computazionale.

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38 Sunto dell'articolo [38]

In questo articolo divulgativo si fa la storia delle ricerche sulla distribuzione dei numeri primi, da Euclide ai giorni nostri, passando per Eulero, Riemann e tanti altri matematici di prima grandezza.

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39 Abstract of paper [39]

We study an extension of Montgomery's pair-correlation conjecture and its relevance in some problems on the distribution of prime numbers.

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40 Abstract of paper [40]

We further study an extension of Montgomery's pair-correlation conjecture and its relevance in some problems on the distribution of prime numbers, in particular, distribution in short intervals.

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41 Abstract of paper [41]

We prove existence of sums of a prime and two squares of primes in very short intervals.

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42 Abstract of paper [42]

We give several asymptotic formulae for the average number of representations of an integer as a sum of a prime and a square. We adapt Hardy & Littlewood's original approach to additive problems with infinite series, since this method is more precise when dealing with short intervals.

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43 Abstract of paper [43]

We give several asymptotic formulae for the average number of representations of an integer as a sum of two prime squares. We adapt Hardy & Littlewood's original approach to additive problems with infinite series, since this method is more precise when dealing with short intervals.

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44 Sunto dell'articolo [44]

Si discutono mezzi "meccanici" per generare la lista dei numeri primi: la macchina di Conway, la formula di Gandhi, il crivello di Eratostene e la formula di Legendre.

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45 Abstract of paper [45]

This paper is a survey of recent results on primes in short intervals: see [30], [39] and [40].

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46 Abstract of paper [46]

Papers on the web

47 Sunto dell'articolo [50]

È il primo di una serie di articoli divulgativi in cui si raccolgono proprietà dei numeri primi che di solito non si trovano nei libri di testo. In particolare, si parla del Teorema Fondamentale dell'Aritmetica, del Crivello di Eratostene e della densità dei primi nella successione dei numeri naturali. In Appendice si danno risultati piú complessi.

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48 Sunto dell'articolo [51]

È il secondo di una serie di articoli divulgativi in cui si raccolgono proprietà dei numeri primi che di solito non si trovano nei libri di testo. Qui parliamo di criteri di primalità ed algoritmi di fattorizzazione, di certificati di primalità, di numeri primi di forma speciale. Anche qui, risultati piú complessi sono dati in Appendice.

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49 Sunto dell'articolo [52]

Si tratta di un breve annuncio del risultato di Goldston, Pintz e Yıldırım a proposito della dimostrazione, lungamente attesa, del fatto che

$$\liminf_{n\to\infty} \frac{p_{n+1} - p_n}{\log p_n} = 0,$$

dove p_n indica l'n-esimo numero primo.

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50 Sunto dell'articolo [53]

Questo articolo contiene il testo di una conferenza divulgativa tenuta per la "Settimana della cultura scientifica e tecnologica" del 2005. Si parla di due esempi di crittogrammi presenti nella letteratura, e precisamente "Lo scarabeo d'oro" di Edgar Allan Poe, e "Viaggio al centro della terra" di Jules Verne. Poi si esamina il metodo di Cesare, uno dei piú antichi metodi crittografici noti in Occidente, e le debolezze di questi sistemi crittografici classici. Infine si esaminano un paio di sistemi crittografici moderni, e se ne studiano le basi matematiche.

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51 Sunto dell'articolo [54]

Questo articolo descrive lo stato dell'arte riguardo la questione degli intervalli fra numeri primi consecutivi, nei due casi di grandi o piccole deviazioni dal comportamento "medio." Dimostriamo alcuni risultati che, pur non essendo i migliori oggi noti, sono pur sempre non banali e illustrano bene le tecniche che si usano in questo campo. Questo articolo ha un livello decisamente superiore e si rivolge a studenti universitari.

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52 Sunto dell'articolo [55]

Si tratta di un resoconto delle idee che hanno portato ai recenti sviluppi nel problema dei "primi gemelli."

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53 Sunto dell'articolo [56]

Si tratta di una spiegazione elementare del risultato di Terence Tao.

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