Abstract of paper [1].

We find zero-free regions and density theorems for the Riemann zeta-function depending on non-trivial estimates for

$$J(x,\theta) := \int_x^{2x} |\psi(t) - \psi(t-\theta t) - \theta t|^2 \mathrm{d}t,$$

uniformly in some range of θ. A Corollary of our main result is the following: Assume that $J(x,\theta) = \mathcal{O}\left(\frac{x^3\theta^2}{F(x\theta)}\right)$, uniformly for $x^{-\beta} \le \theta \le 1$, where F is a positive, increasing function which is unbounded as $x \to +\infty$, such that $F(x) = \mathcal{O}(x^\varepsilon)$ for every $\varepsilon > 0$, and that $\beta \in (0,1)$. Then the Riemann zeta-function does not vanish in the region

$$\sigma > 1 - C\frac{\log F(t)}{\log t},$$

where C is some absolute, positive constant. The proof is based on Turán's power sum method.

References

[1] A. Zaccagnini. A conditional density theorem for the zeros of the Riemann zeta-function. *Acta Arithmetica*, 93:293–301, 2000.