Abstract of paper [1].

Let Λ be the von Mangoldt function and

$$R(n) = \sum_{h+k=n} \Lambda(h)\Lambda(k).$$

Let further N, H be two integers, $N \geq 2$, $1 \leq H \leq N$, and assume that the Riemann Hypothesis holds. Then

$$\sum_{n=N-H}^{N+H} R(n)\Big(1 - \frac{|n-N|}{H}\Big) = HN - \frac{2}{H}\sum_{\rho} \frac{(N+H)^{\rho+2} - 2N^{\rho+2} + (N-H)^{\rho+2}}{\rho(\rho+1)(\rho+2)}$$
$$+ \Big(N\Big(\log\frac{2N}{H}\Big)^2 + H(\log N)^2 \log(2H)\Big),$$

where $\rho = 1/2 + i\gamma$ runs over the non-trivial zeros of the Riemann zeta-function $\zeta(s)$.

References

[1] A. Languasco and A. Zaccagnini. Cesàro average in short intervals for Goldbach numbers. *Proc. Amer. Math. Soc.*, 145(10):4175–4186, 2017.