**Abstract of paper [3].**

In their third paper of the "Partitio Numerorum" series, Hardy & Littlewood conjectured that for $k = 2, 3$ and for $n \to \infty$, $n \neq m^k$, there is an asymptotic formula for $r_k(n) := |\{(m, p) \colon\ n = m^k + p,\ p \text{ is a prime}\}|$ and that, in particular, $r_k(n) \to \infty$ if $n \to \infty$, $n \neq m^k$. For $k \geq 2$ let $E_k(X) := |\{n \leq X \colon\ r_k(n) = 0\}|$ be the number of exceptions to the "weak" conjecture $r_k(n) \geq 1$ for $n \geq n_0(k)$, $n$ not a power. We prove that there exists $\delta = \delta(k) > 0$ such that $E_k(X) \ll X^{1-\delta}$, that $E_k(X + H) - E_k(X) \ll H(\log X)^{-A}$ for $X^{\frac{7}{12}(1-1/k)+\epsilon} \leq H \leq X$, and also give estimates for the number of integers for which the asymptotic formula actually holds. Furthermore, we give explicit estimates for $\delta(k)$ under the Generalized Riemann Hypothesis. This paper contains in part [2, 1] and a sketch of the circle method, as used in these problems.

# References

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