Abstract of paper [1].

Let Λ be the von Mangoldt function and $R(n) = \sum_{h+k=n} \Lambda(h)\Lambda(k)$ be the counting function for the Goldbach numbers. Let $N \geq 2$ and assume that the Riemann Hypothesis holds. We prove that

$$\sum_{n=1}^{N} R(n) = \frac{N^2}{2} - 2\sum_{\rho} \frac{N^{\rho+1}}{\rho(\rho+1)} + O\big(N(\log N)^3\big),$$

where $\rho = 1/2 + i\gamma$ runs over the non-trivial zeros of the Riemann zeta-function $\zeta(s)$.

References

[1] A. Languasco and A. Zaccagnini. The number of Goldbach representations of an integer. *Proc. Amer. Math. Soc.*, 140:795–804, 2012.