Abstract of paper [1].

We give a new estimate for the integral moments of primes in short intervals of the type $(p, p+h]$, and prove that for every $\lambda > 1/2$ there exists a positive proportion of primes $p \leq X$ such that the interval $(p, p+\lambda \log X]$ contains at least a prime number. We improve Cheer and Goldston's result (1987) on the size of real numbers $\lambda > 1$ with the property that there is a positive proportion of integers $m \leq X$ such that the interval $(m, m+\lambda \log X]$ contains no primes.

References

[1] D. Bazzanella, A. Languasco, and A. Zaccagnini. Primes in logarithmic intervals. *Trans. Amer. Math. Soc.*, 362:2667–2684, 2010.