Abstract of paper [1].

Let

$$G(x;q,a) := \max_{\substack{p_n \le x \\ p_n \equiv p_{n+1} \equiv a \mod q}} (p_{n+1} - p_n),$$

where $(a,q) = 1$ and p_n, p_{n+1} are consecutive primes in the arithmetic progression $a \mod q$. We prove the following result: let C be any fixed positive number, $C < 1$. Then, uniformly for

$$1 \le q \le \exp\exp\left\{C \log\log x \frac{\log\log\log\log x}{\log\log\log x}\right\}$$

we have:

$$G(x;q,a) \ge \big(e^\gamma + o_C(1)\big)\phi(q) \log x \log\log x \frac{\log\log\log\log x}{(\log\log\log x)^2},$$

where γ is Euler's constant. The o-symbol may depend on C.

References

[1] A. Zaccagnini. A note on large gaps between consecutive primes in arithmetic progressions. *J. Number Theory*, 42:100–102, 1992.